CMPU 240 · Theory of Computation

# Relating DFAs and NFAs

5 February 2026

## Assignment 1

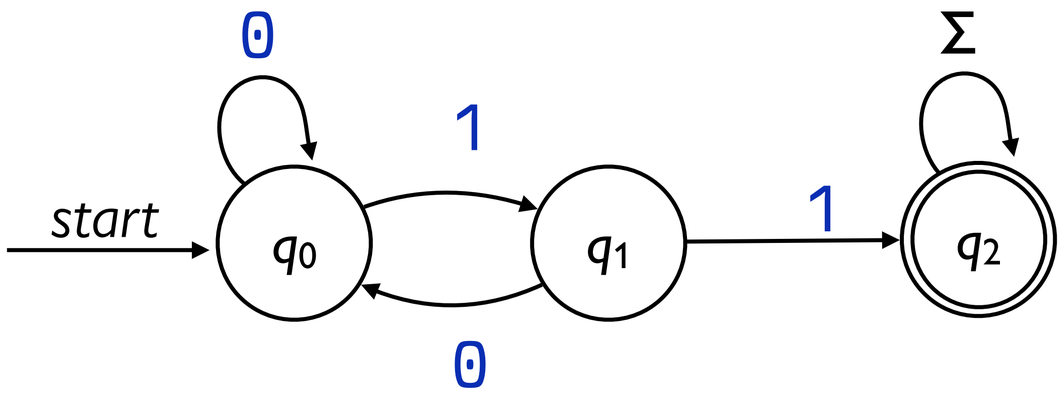
Corrections due today

## Assignment 2

Out today; due on Tuesday

# Where are we?

We can describe a DFA with a state transition diagram,



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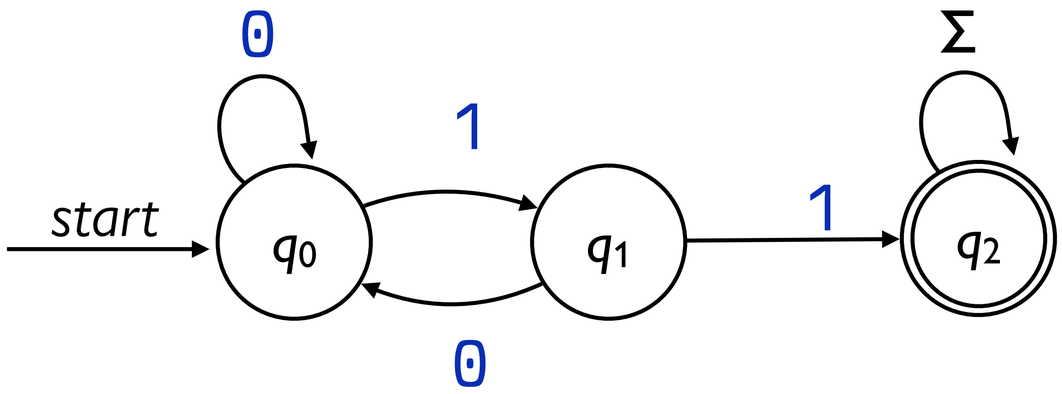



or, equivalently,

| | State | 0 | 1 |
|---|---|---|---|
| → | $q_0$ | $q_0$ | $q_1$ |
| | $q_1$ | $q_0$ | $q_2$ |
| * | $q_2$ | $q_2$ | $q_2$ |

More formally, a DFA is represented as a five-tuple $(Q, \Sigma, \delta, q_0, F)$ where

$Q$ is a finite set of *states*,

$\Sigma$ is the *alphabet*, a finite set of input symbols,

$\delta$: $Q \times \Sigma \rightarrow Q$ is the *transition function*,

$q_0 \in Q$ is the *start state*, and

$F \subseteq Q$ is a set of zero or more *accept states*.

If $D$ is a DFA, the *language of D*, denoted $L(D)$, is

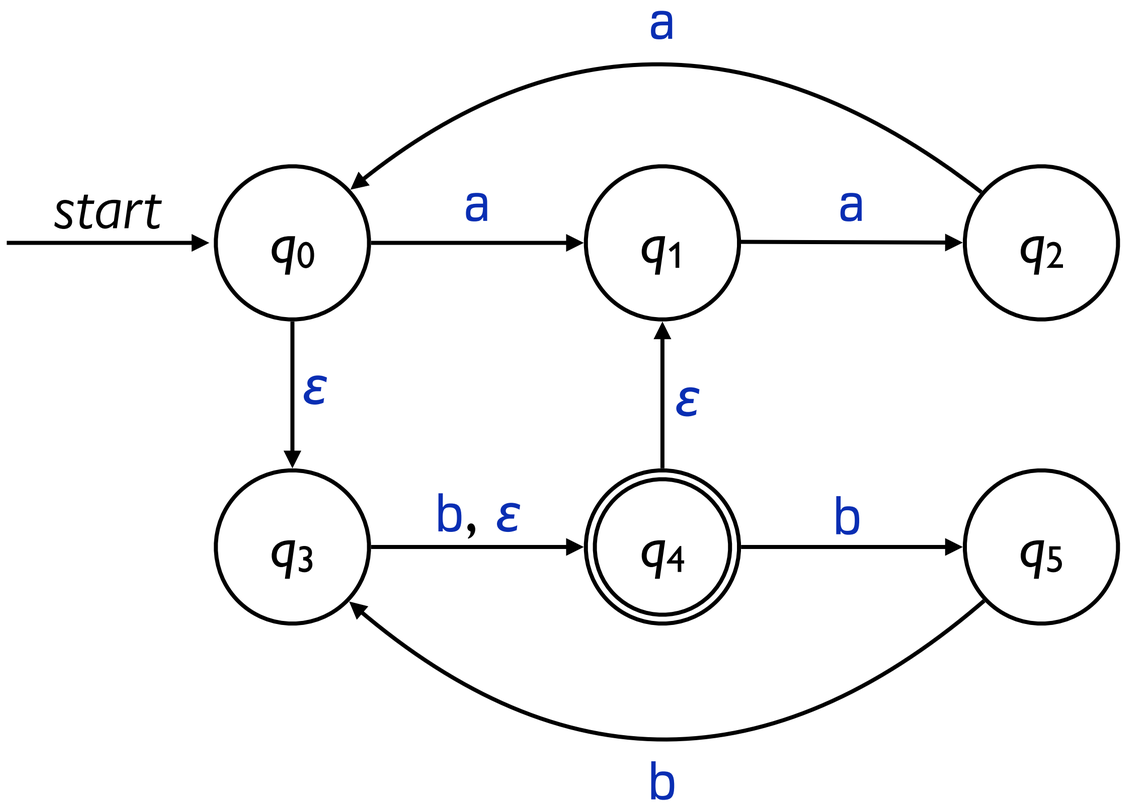
$$\{w \in \Sigma^* \mid D \text{ accepts } w\}.$$

A language is a *regular language* if there exists a DFA $D$ such that $L(D) = L$.

A *nondeterministic finite automaton* (NFA) can is like a DFA, but it can have missing transitions or multiple transitions defined on the same input symbol.

An NFA accepts if *any possible series of choices* leads to an accept state.

NFAs can also have a special type of transition called an *ε-transition*:




An NFA may follow any number of ε-transitions at any time without consuming any input.

Formally, an NFA is defined like a DFA:

$$N = (Q, \Sigma, \delta, q_0, F)$$

Except instead of

$$\delta\colon Q \times \Sigma \to Q,$$

we have

$$\delta\colon Q \times (\Sigma \cup \{\varepsilon\}) \to \wp(Q).$$

An NFA can be thought of as a DFA that can be in many states at once.

At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.

The NFA accepts if *any* of the states that are active at the end are accept states. Otherwise, it rejects.

# Just how powerful *are* NFAs?

*NFAs must be at least as powerful as DFAs.*

Any language that can be recognized by a DFA can be recognized by an NFA.

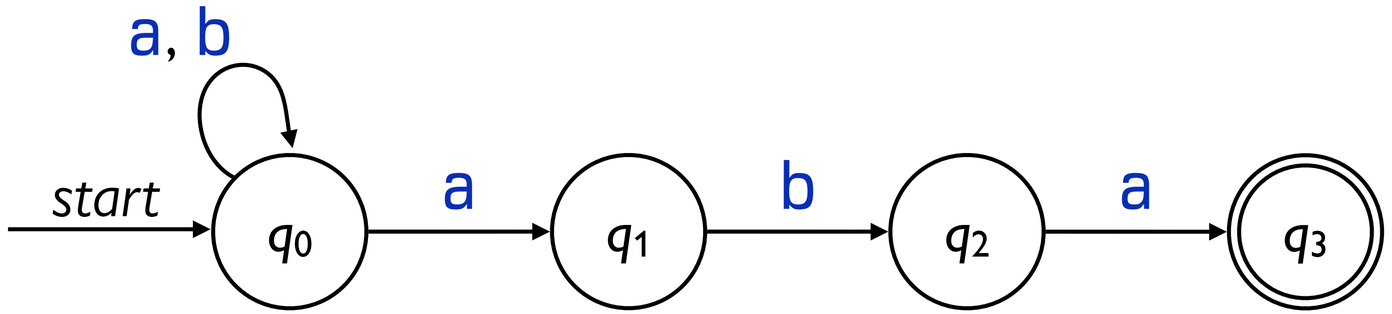
Why? Essentially, every DFA already *is* an NFA – just one that doesn't exploit nondeterminism.

Can every language recognized by an NFA also be recognized by a DFA?

Can every language recognized by an NFA also be recognized by a DFA?

Yes! While NFAs *seem* more powerful, they're not.

*Thought experiment*: How could you simulate an NFA in software?


a, b
start
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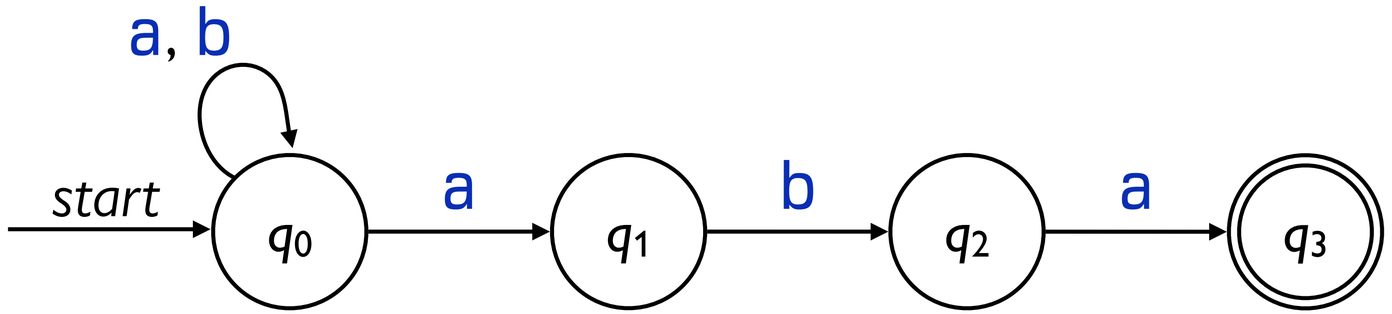
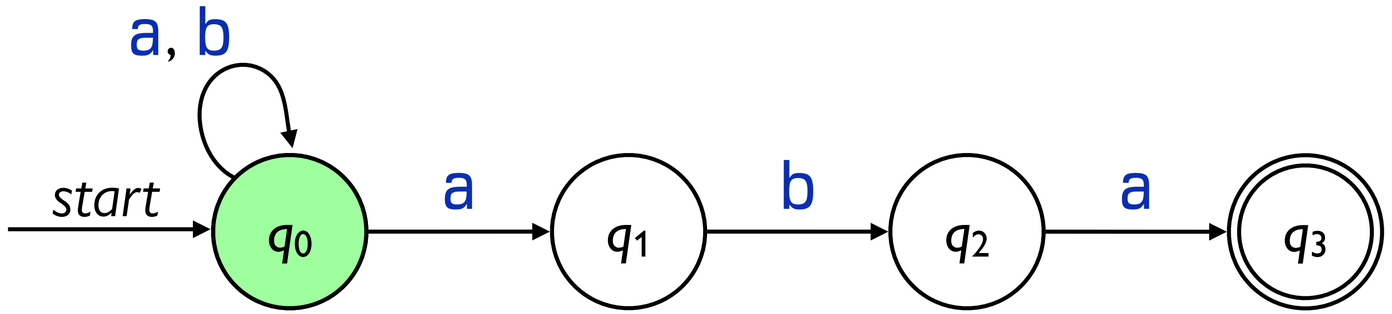
$q_1$
b
$q_2$
a
$q_3$




a b a b a

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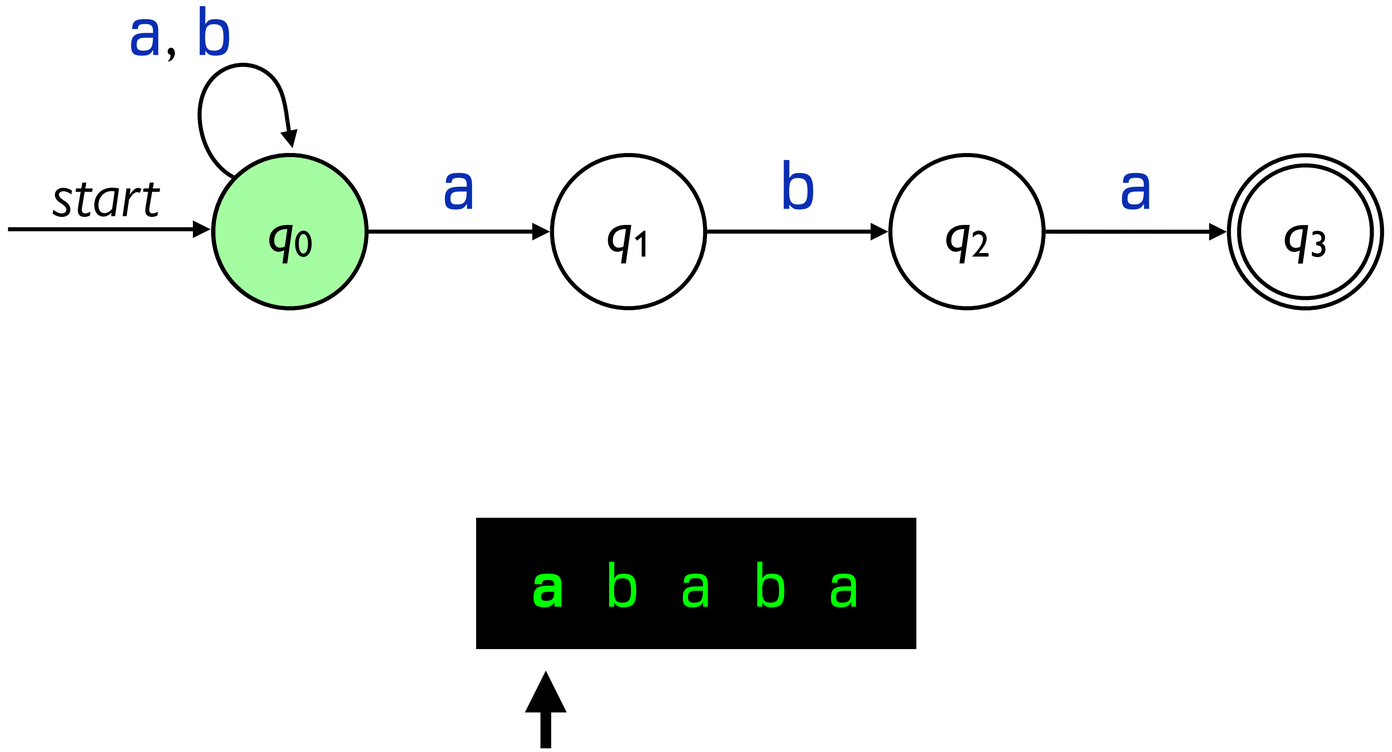
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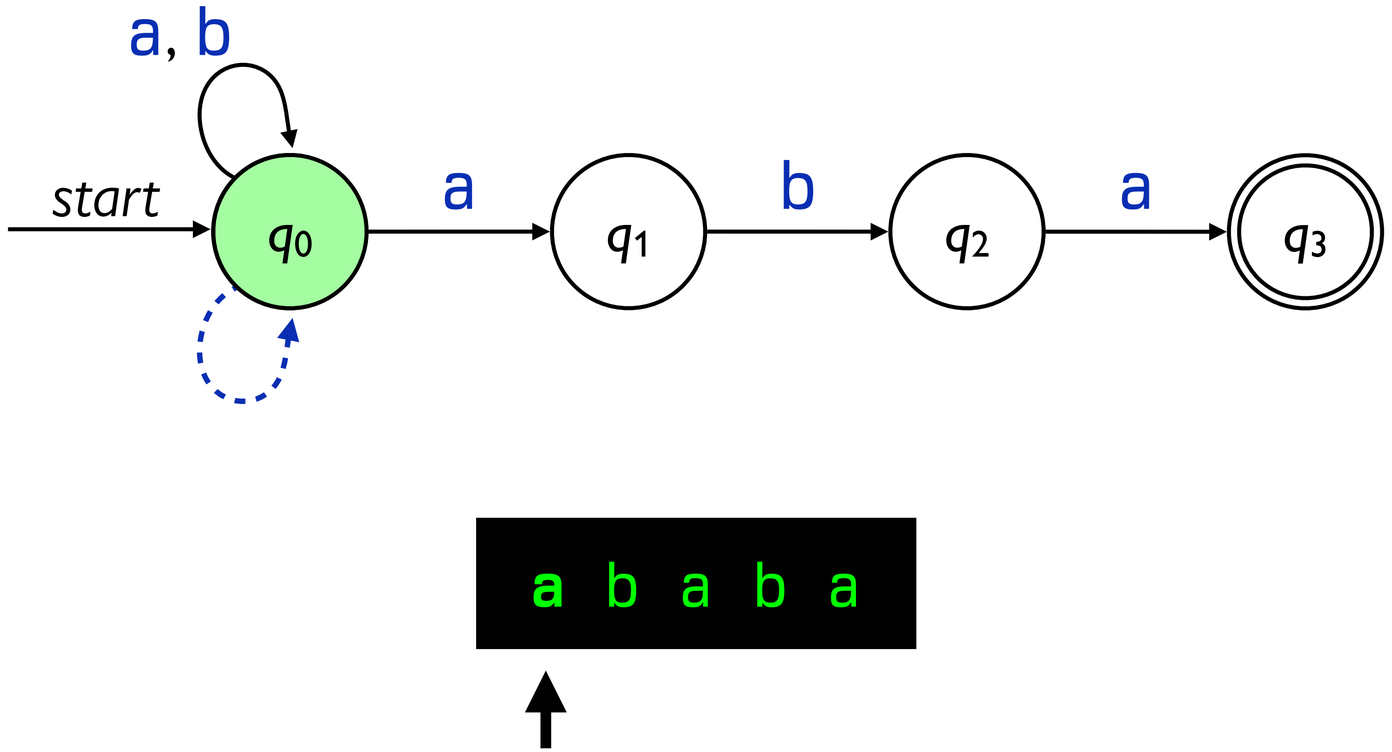
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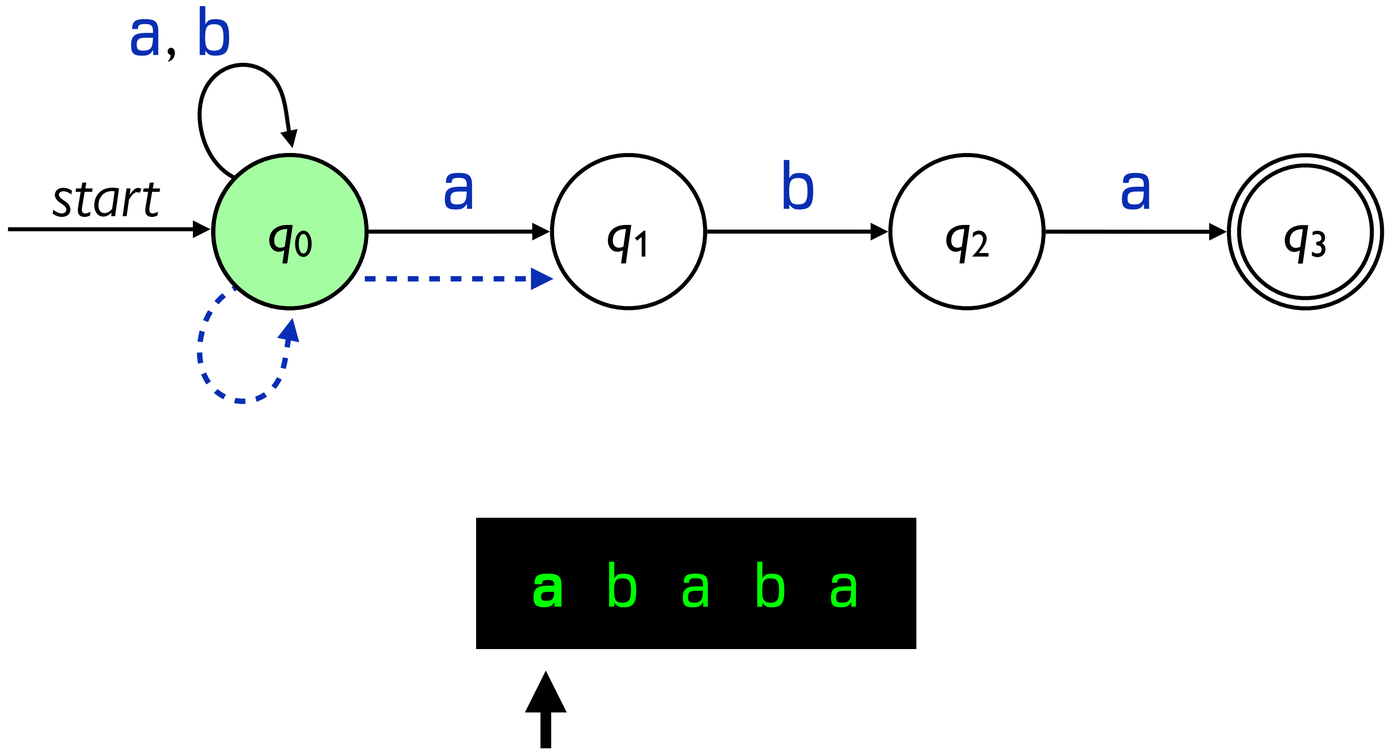
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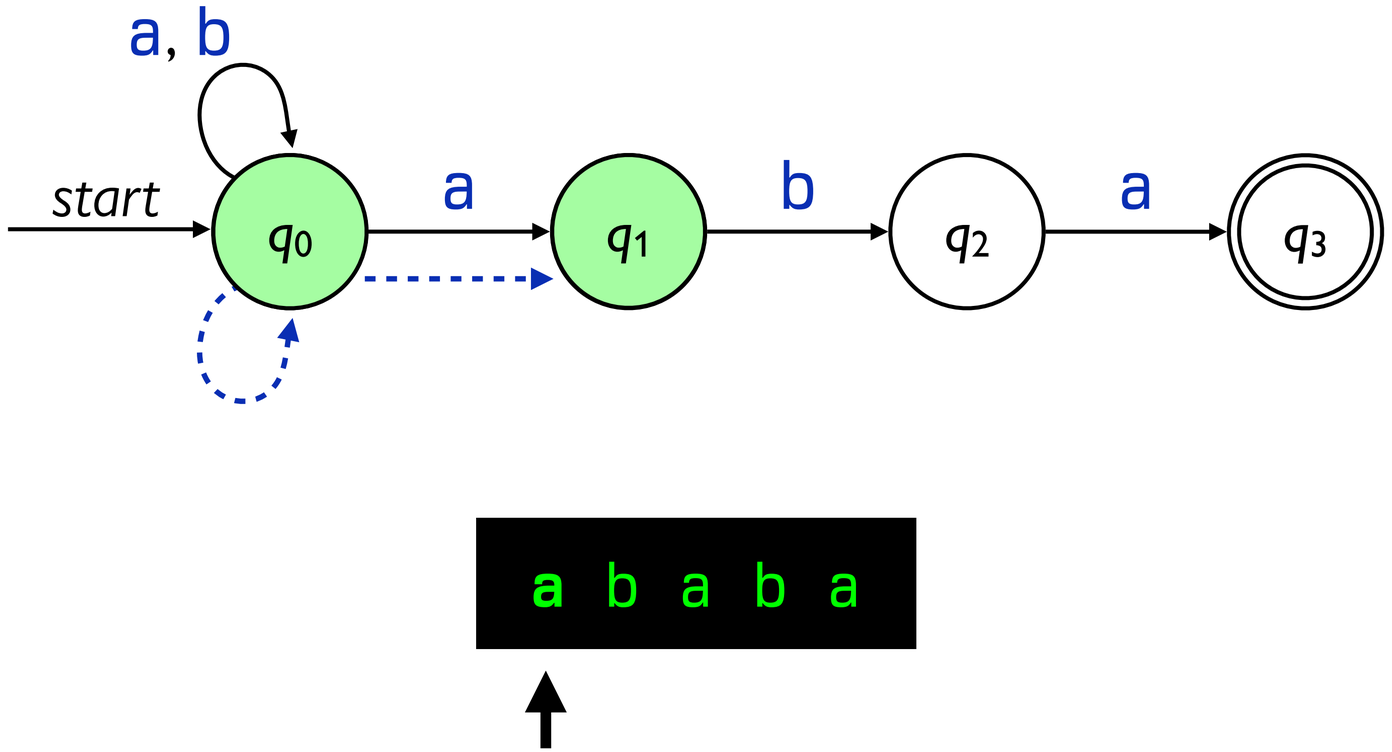
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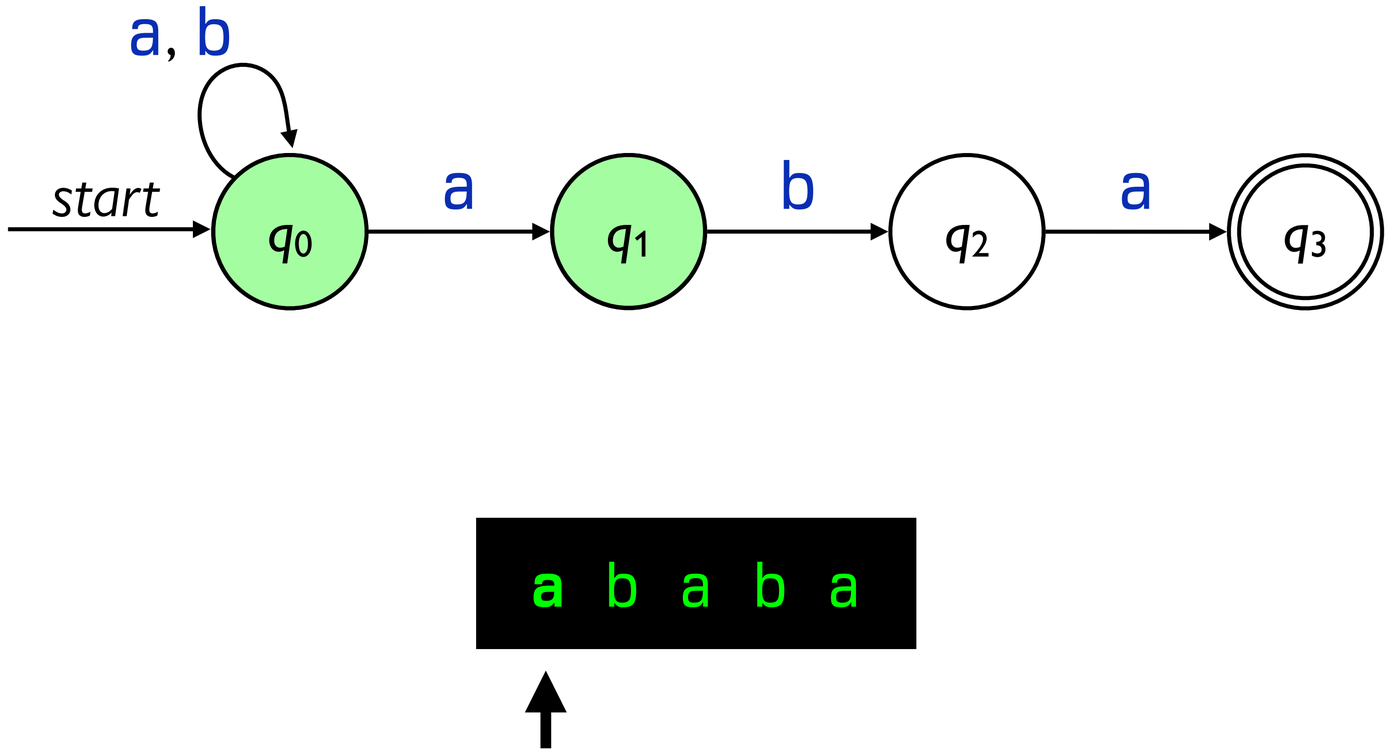
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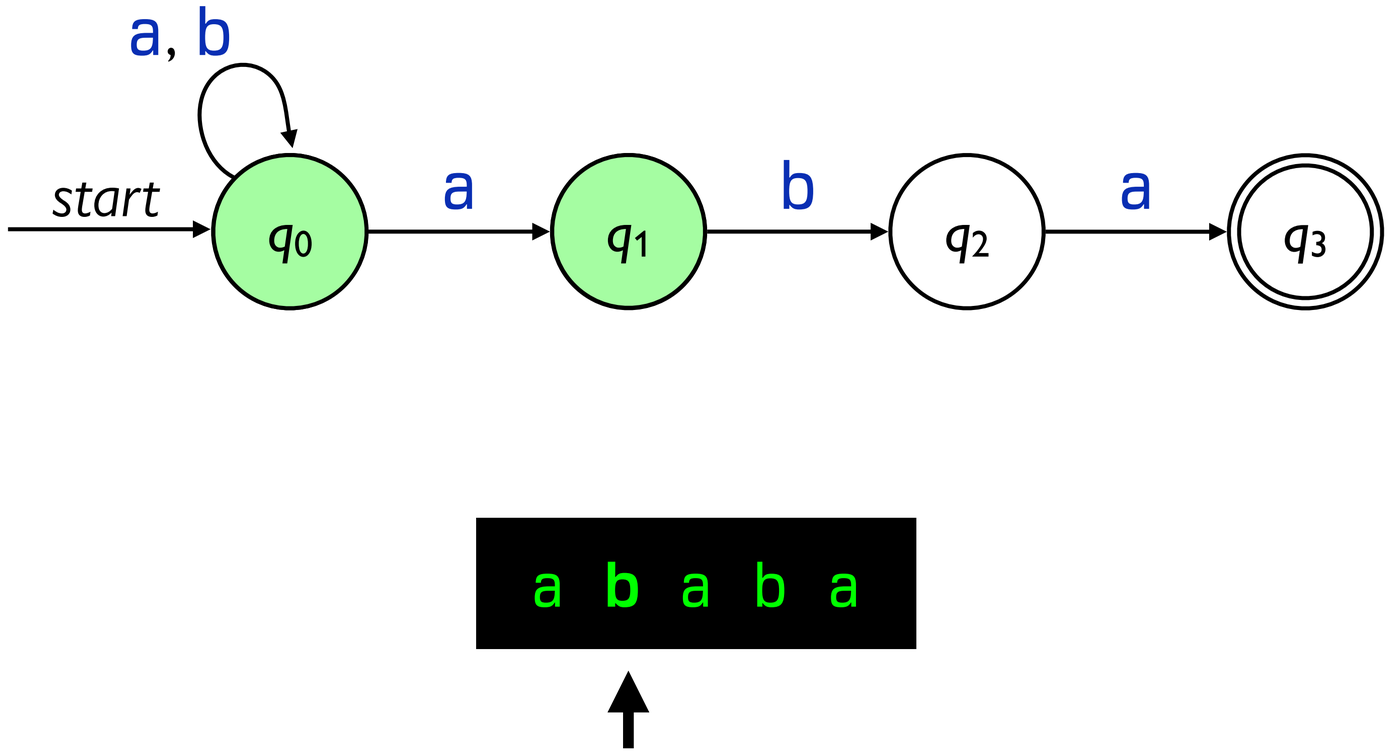
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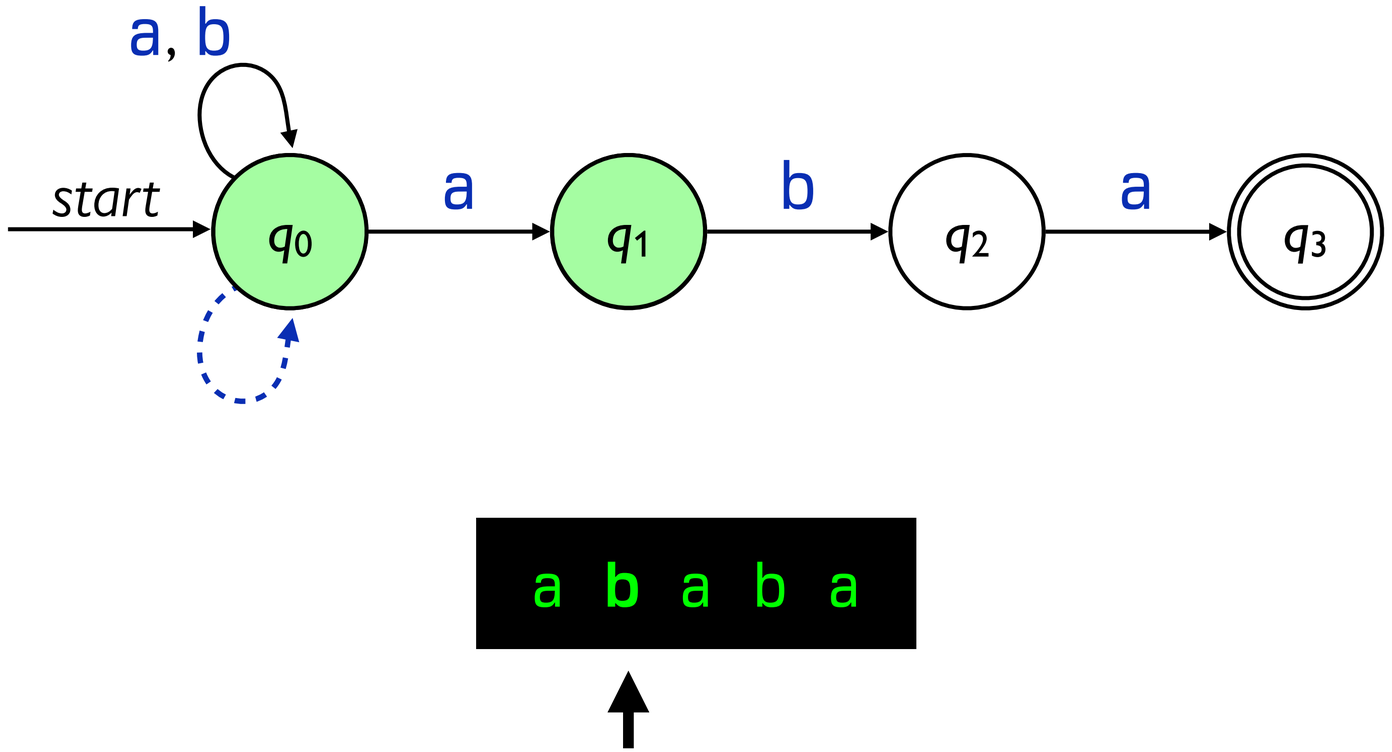
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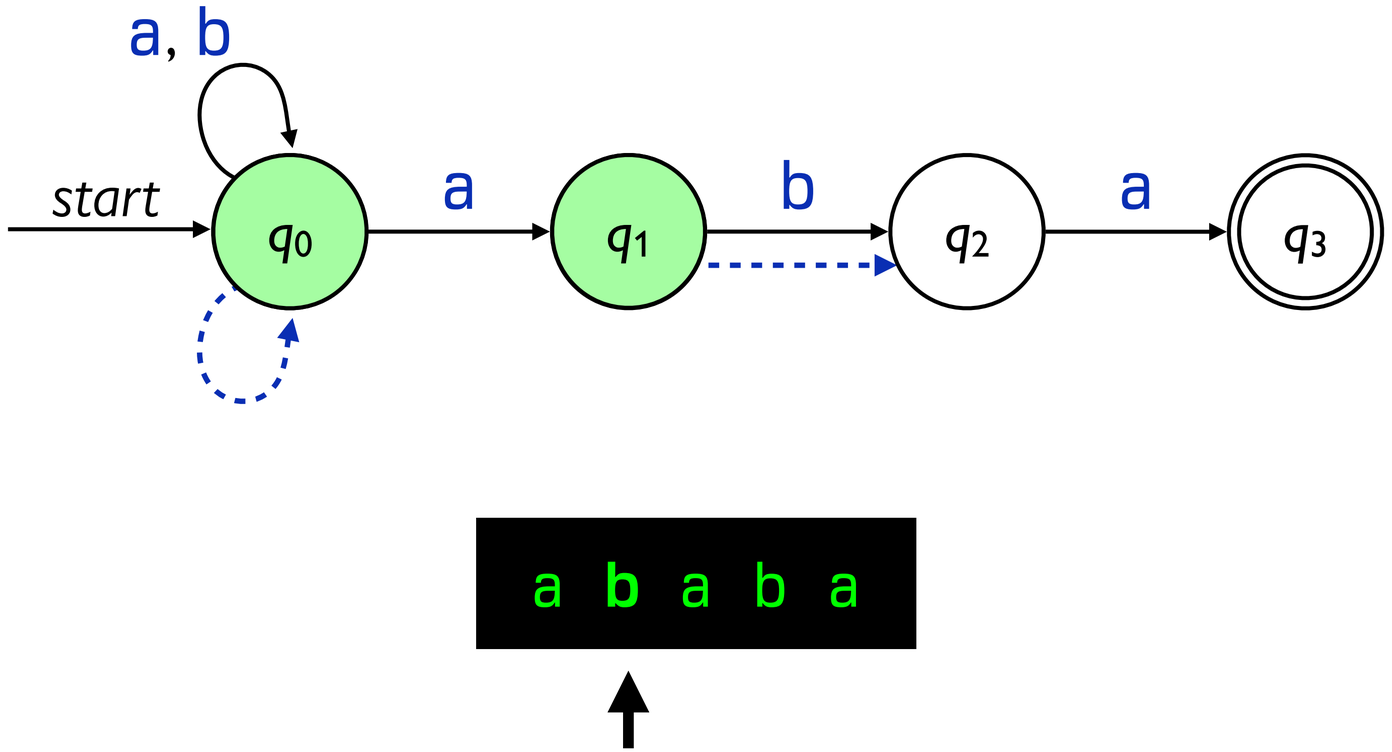
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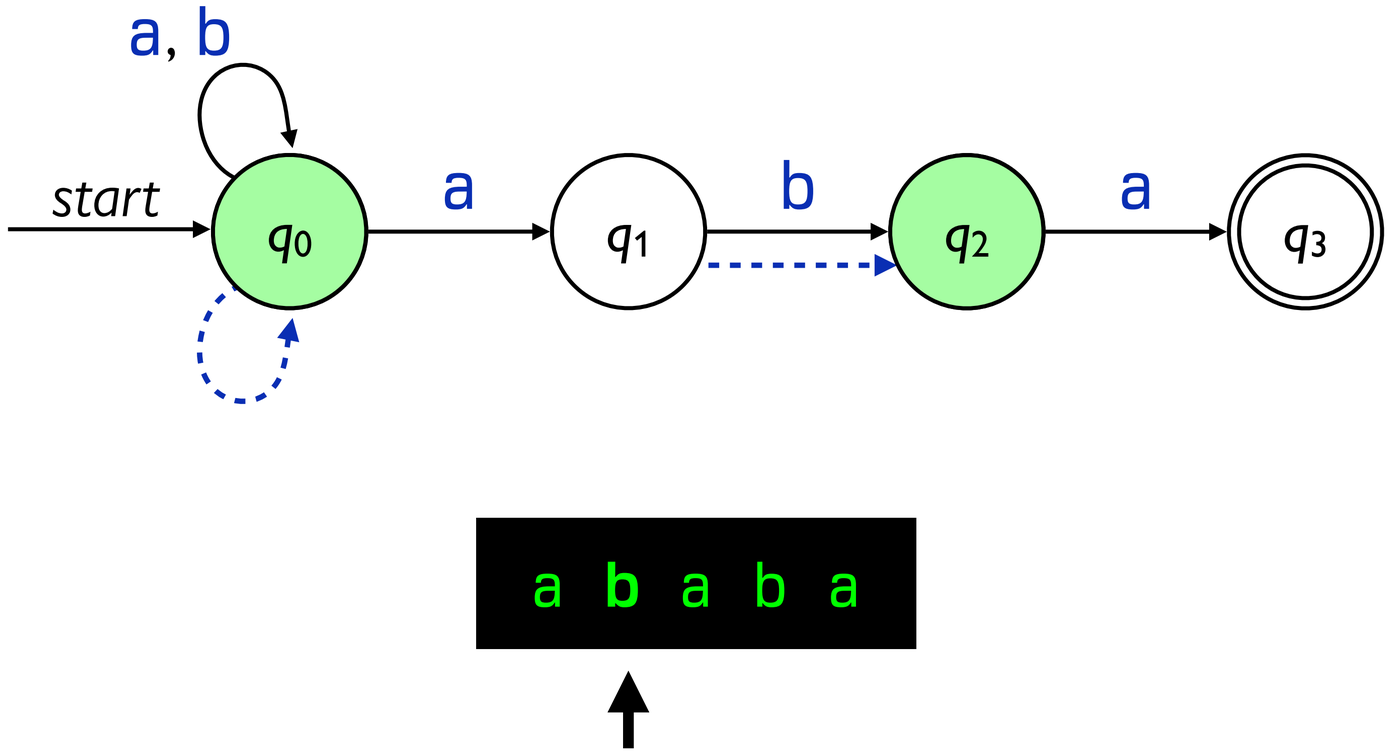
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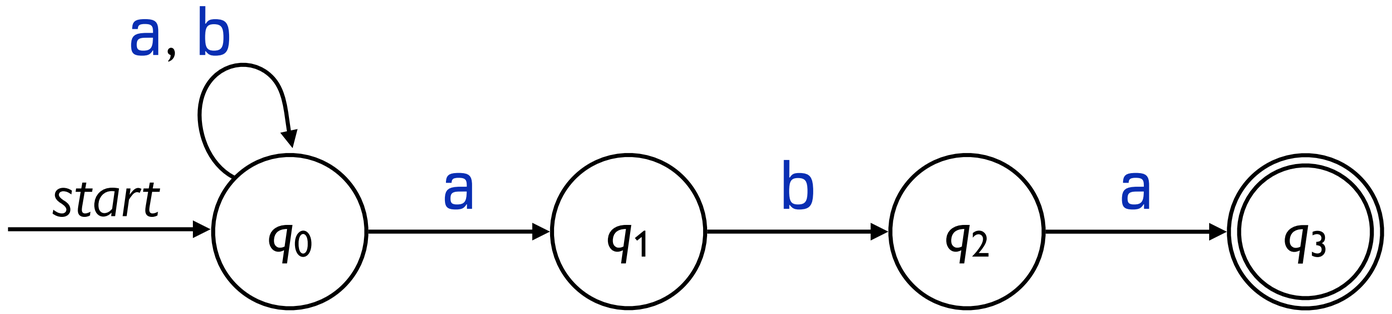


| State | a |
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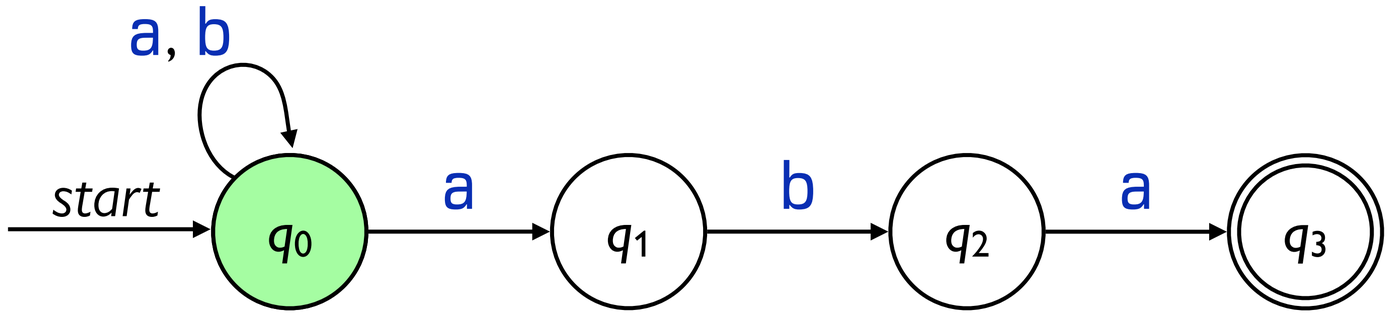
| → $\{q_0\}$ | |




| State | a |
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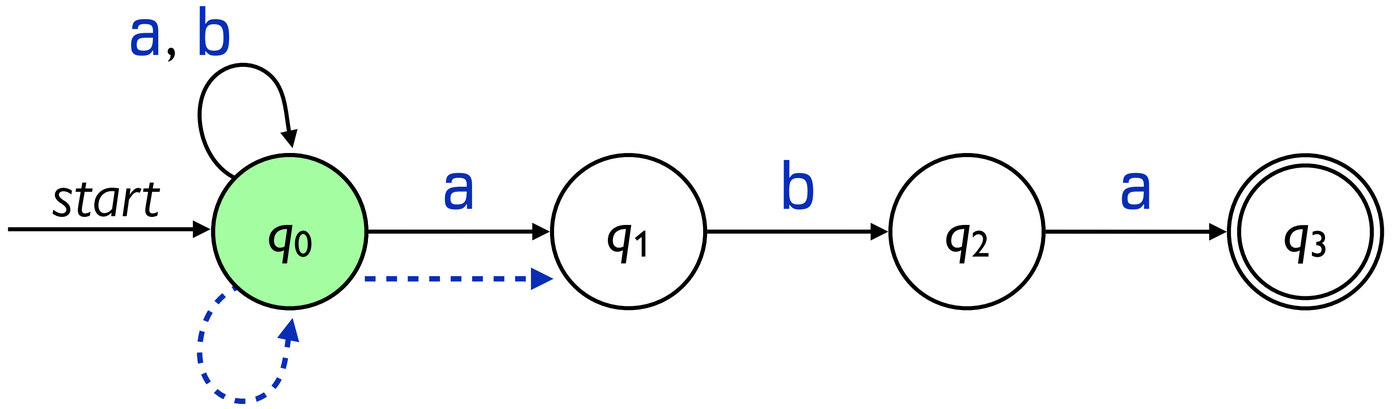


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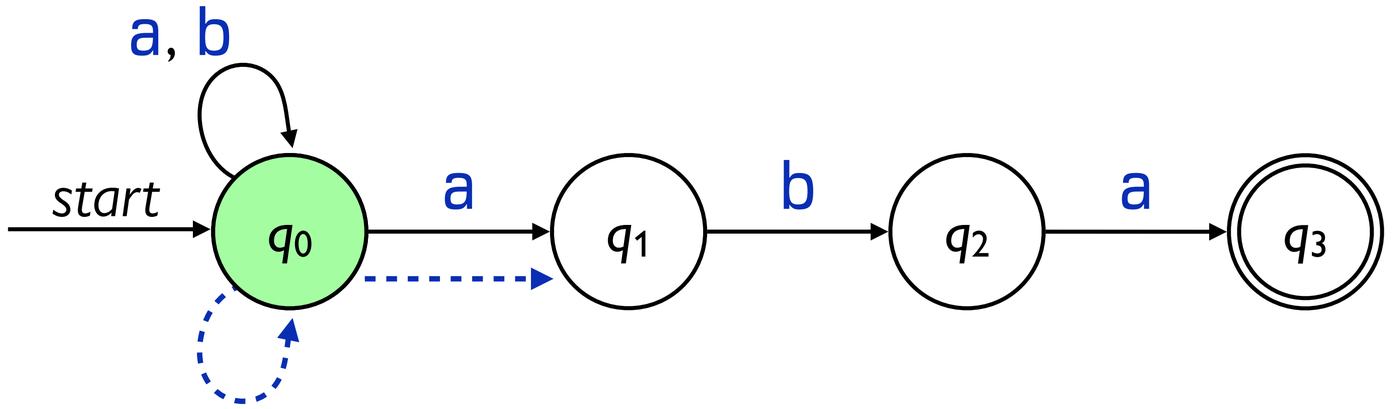


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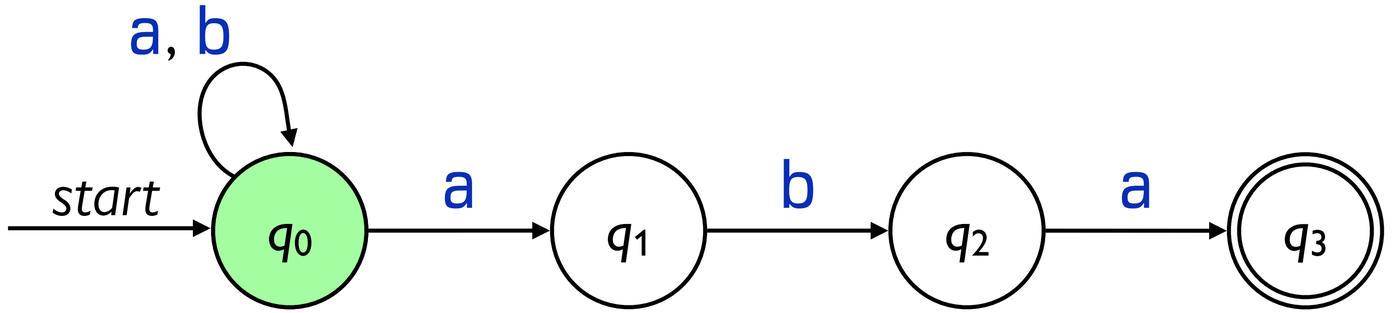


| | State | a | b |
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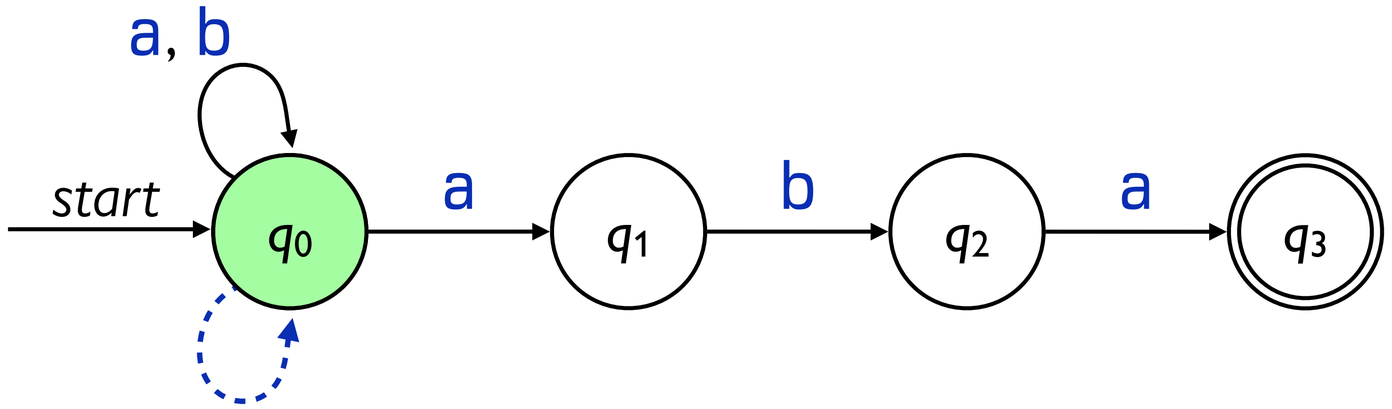
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | |




| | State | a | b |
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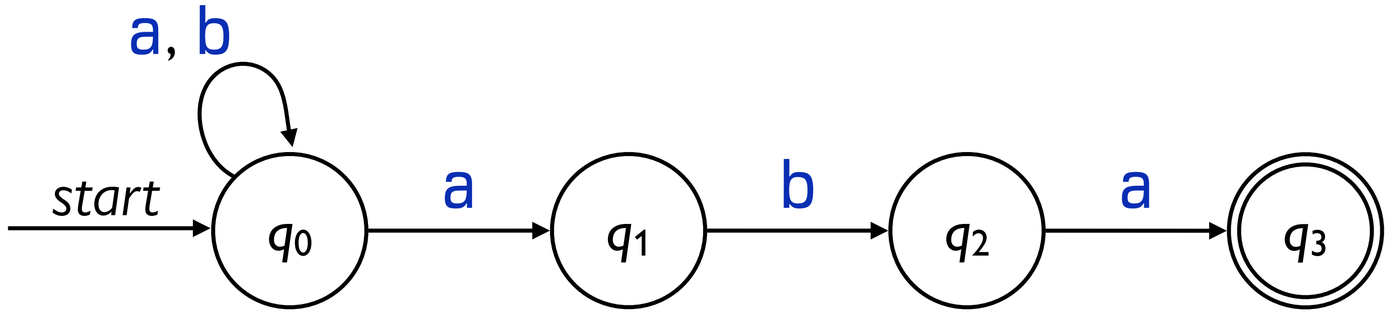
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |

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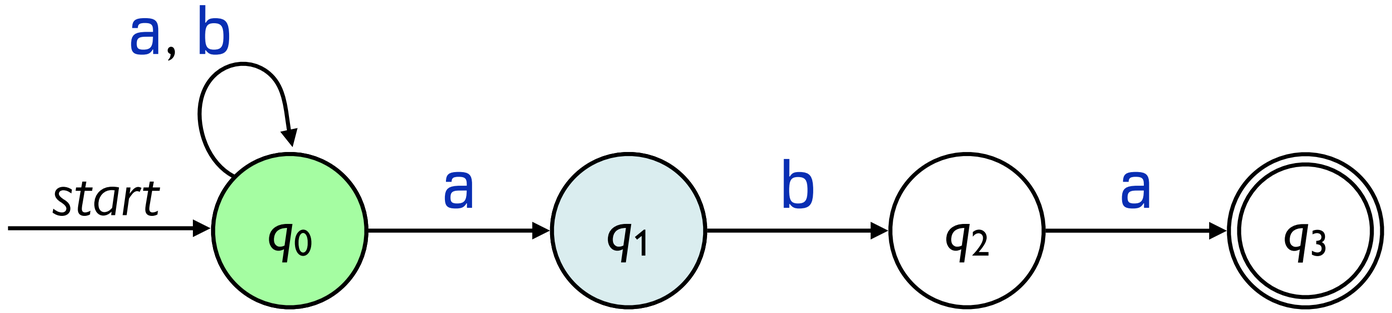


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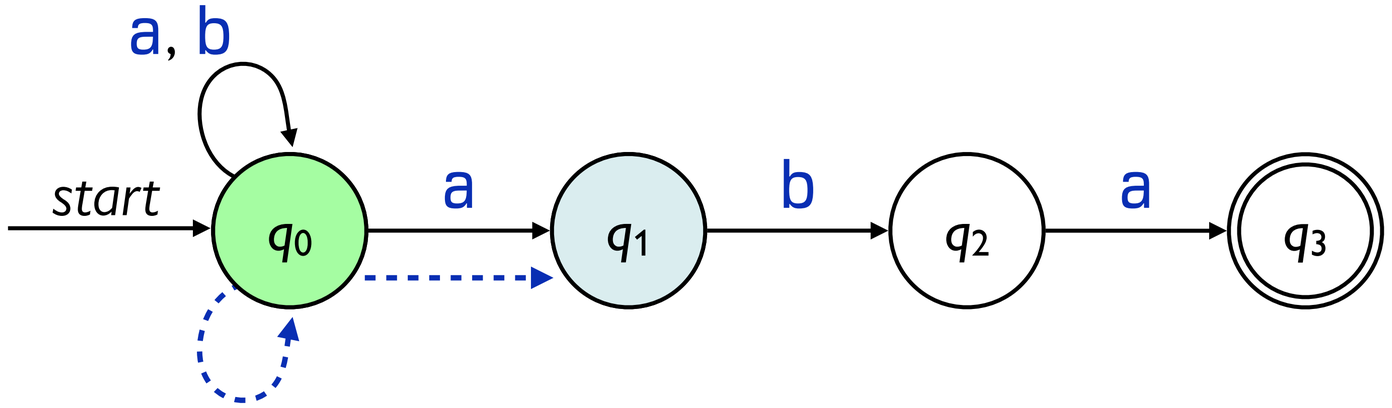


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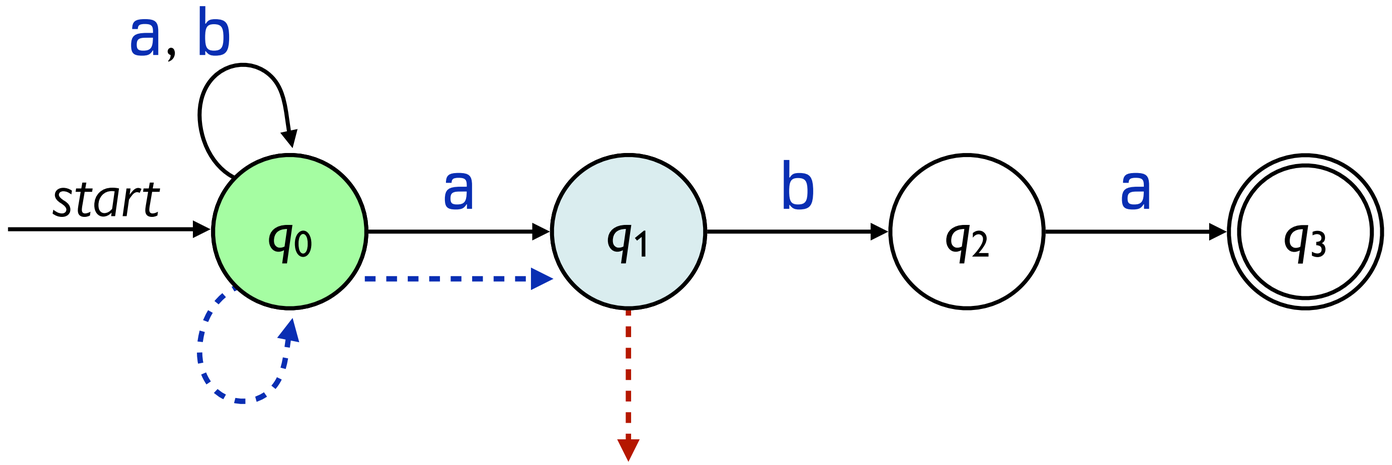


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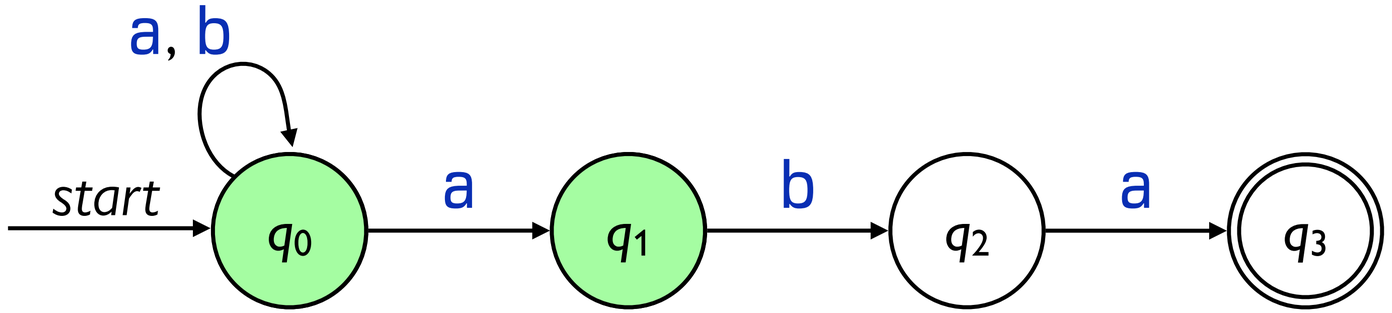


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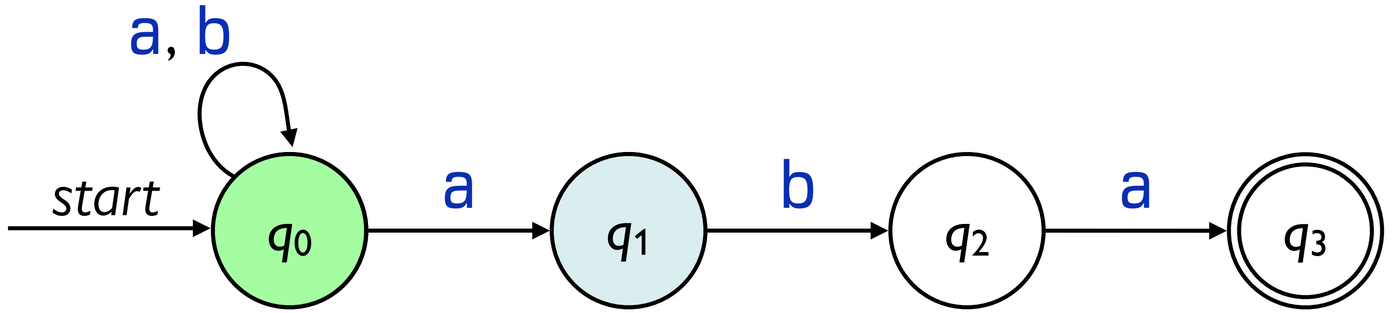


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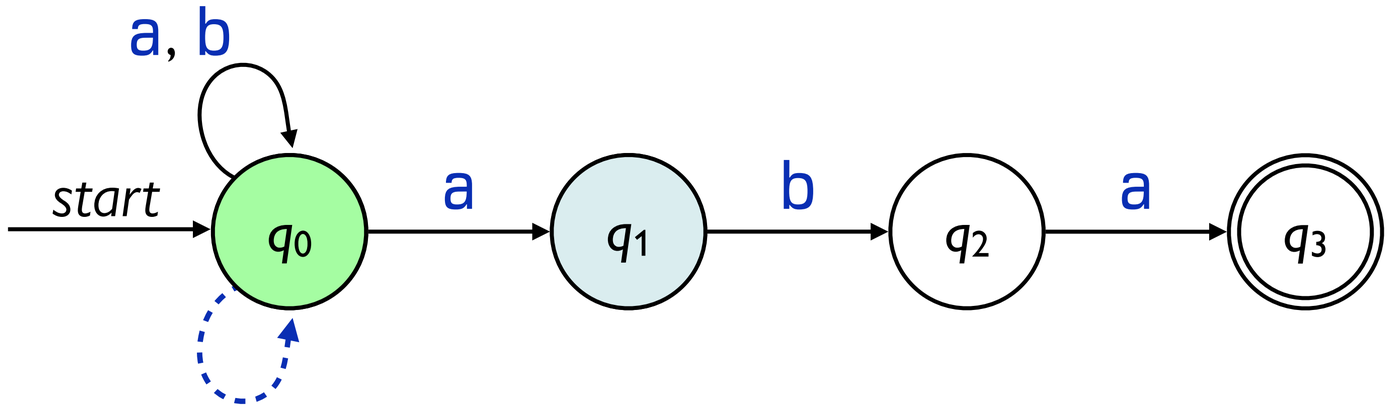


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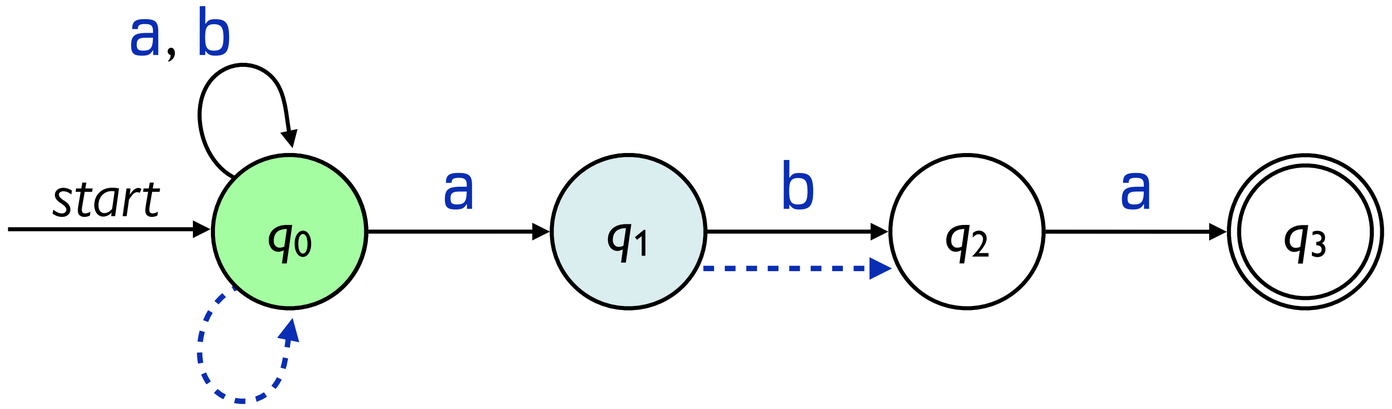


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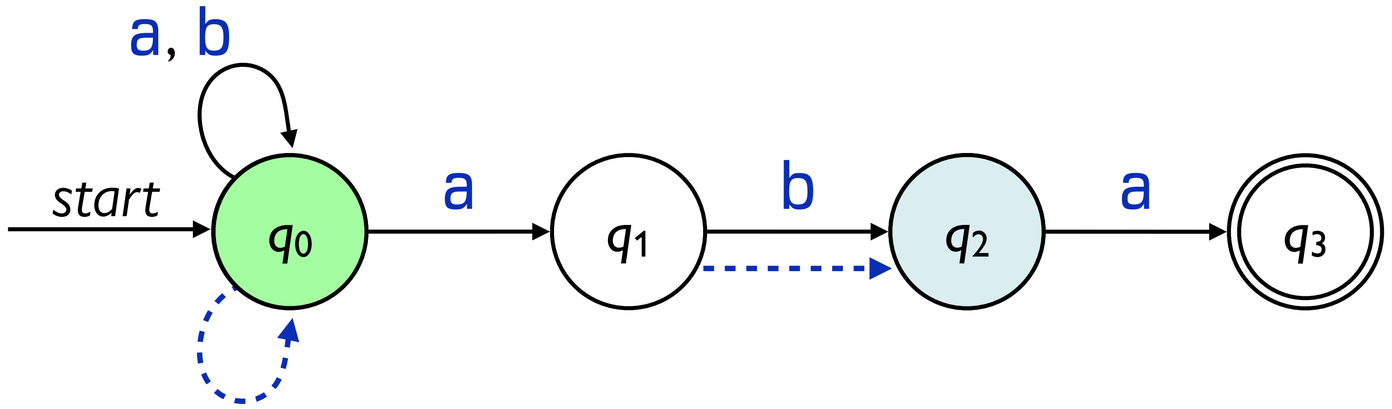


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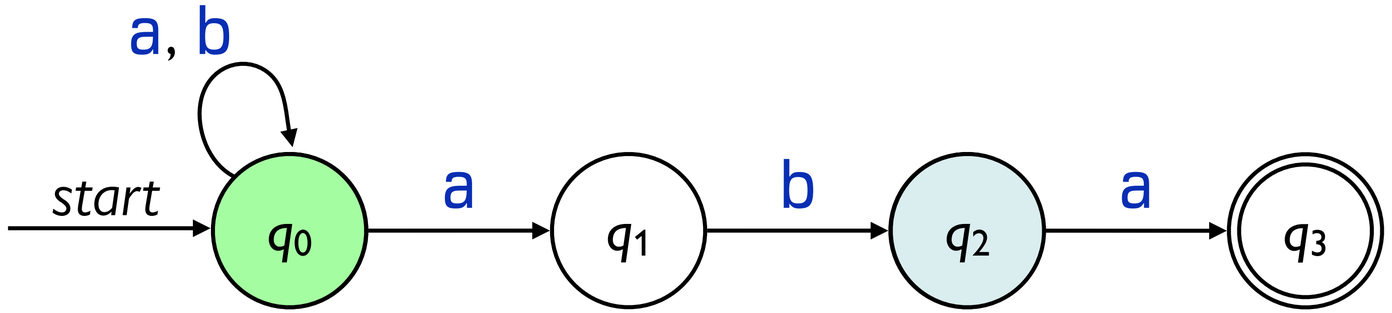


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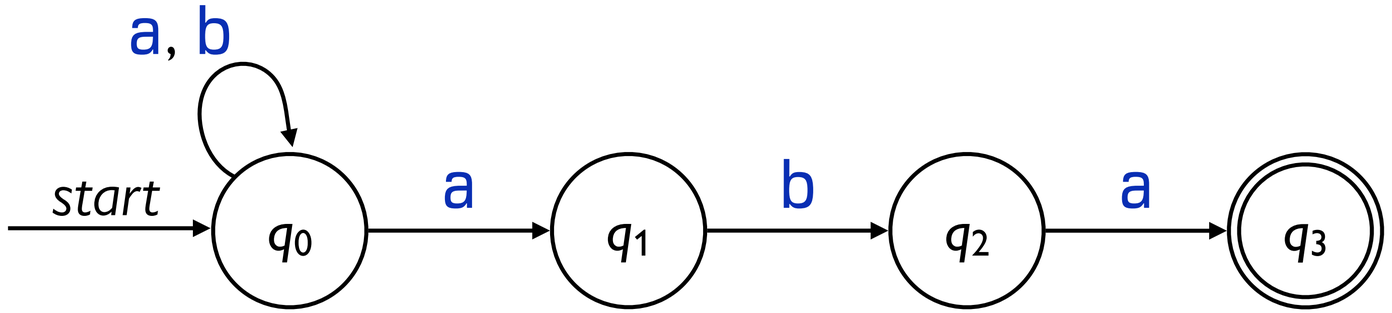


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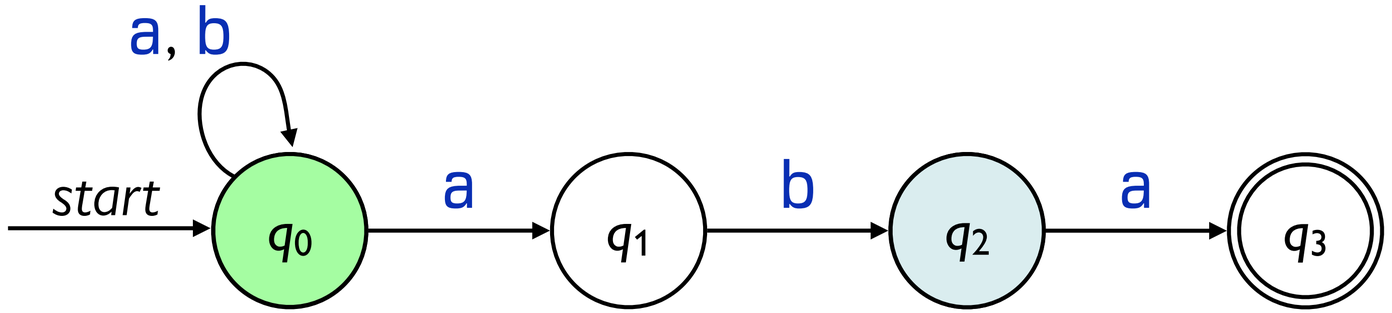


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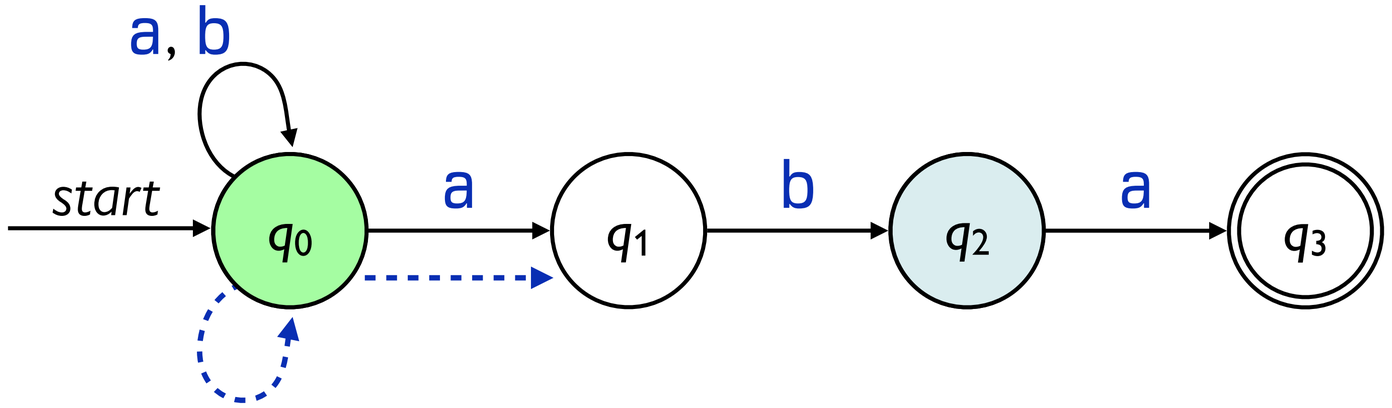


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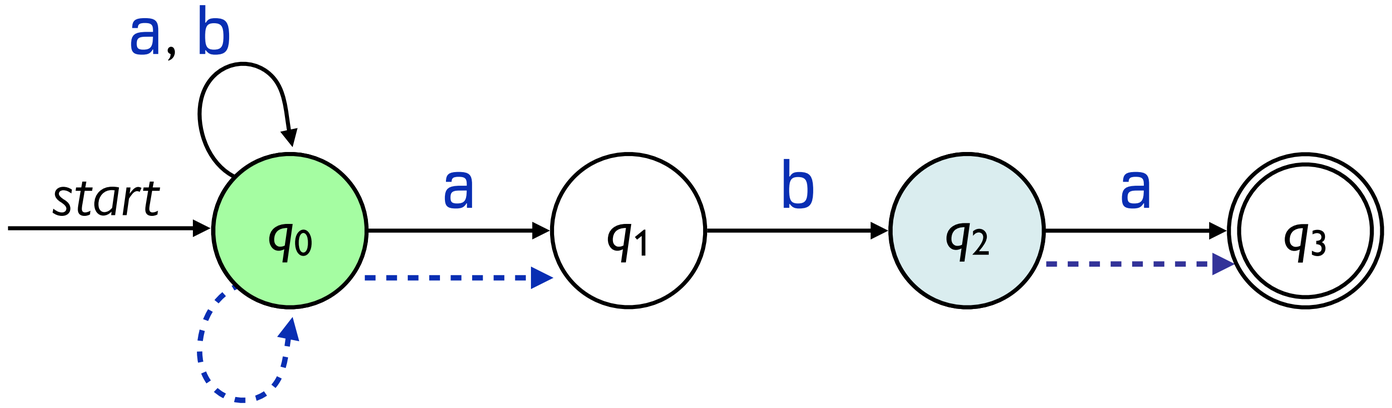


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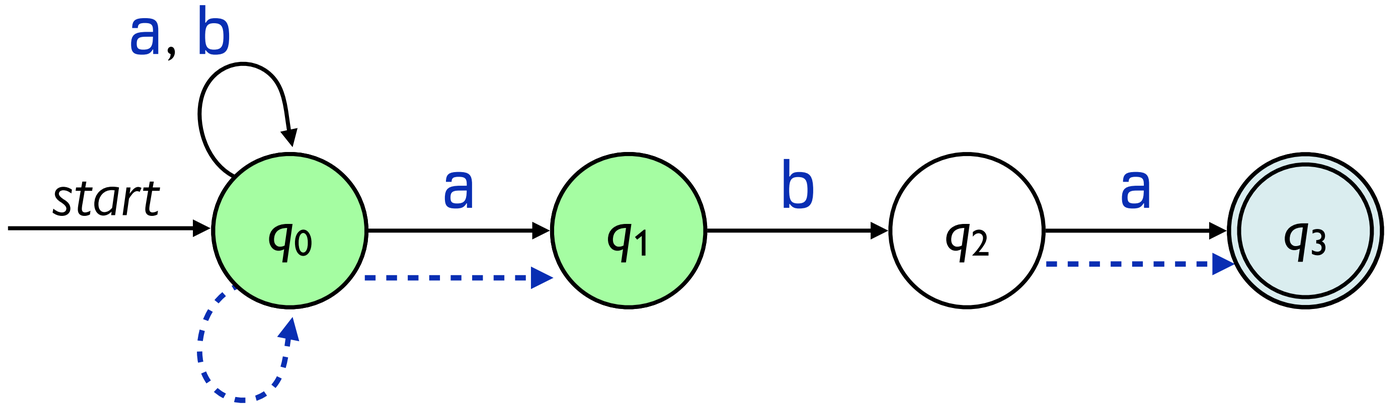


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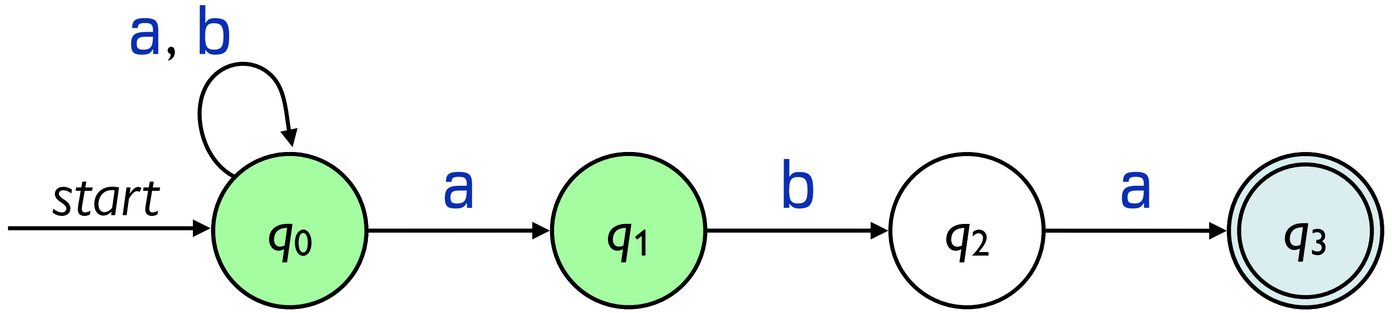


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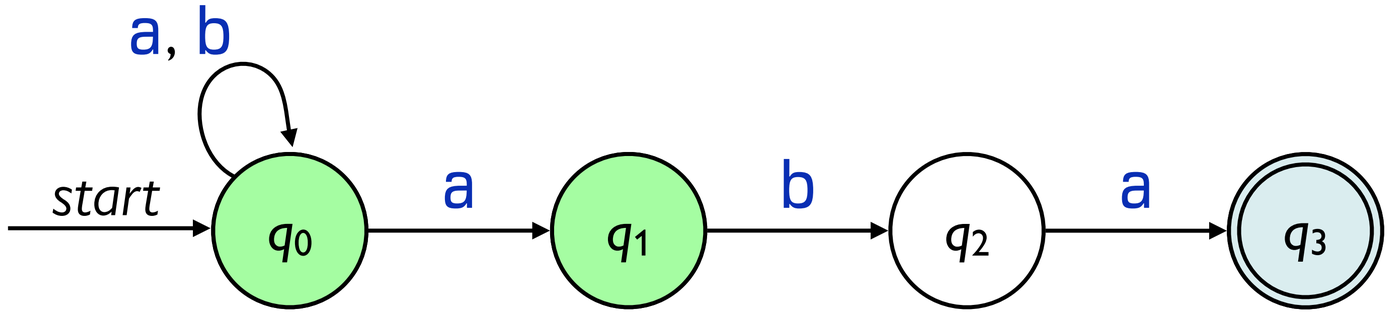


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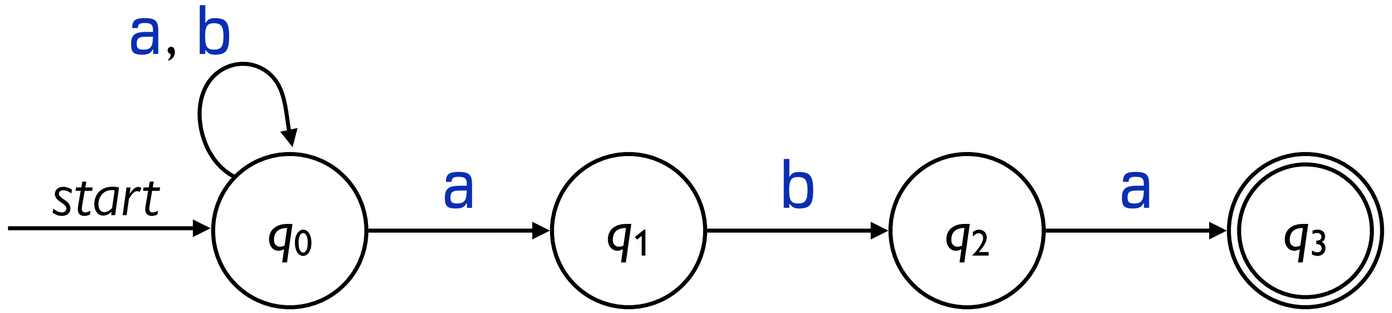


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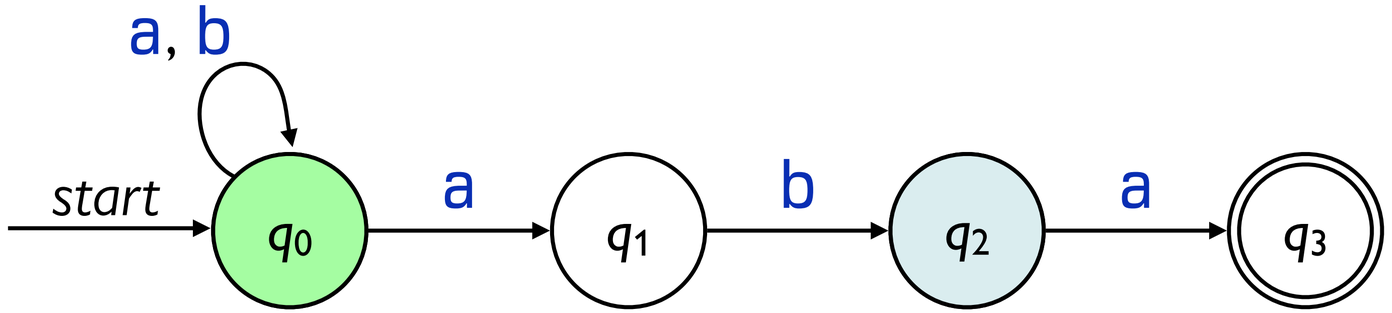


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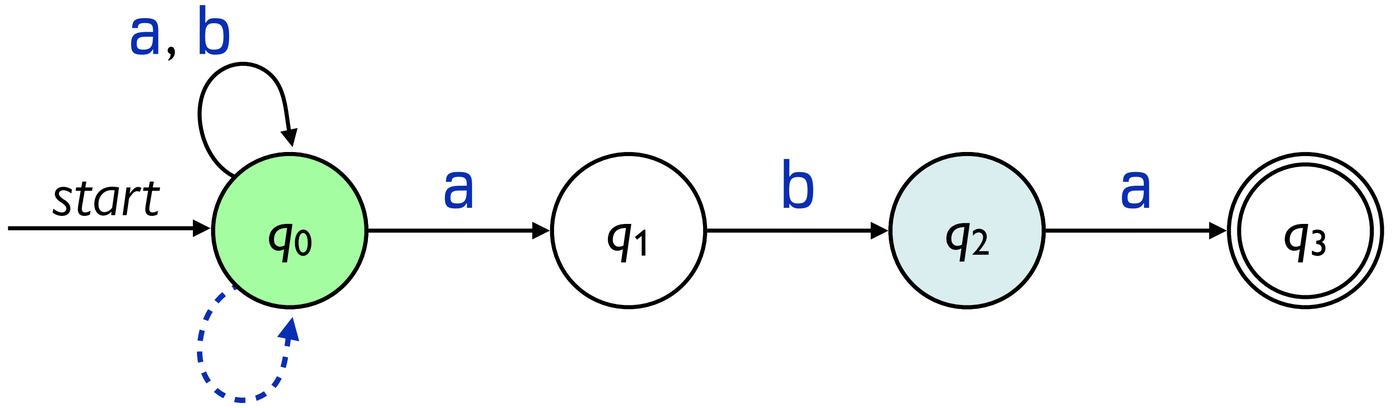
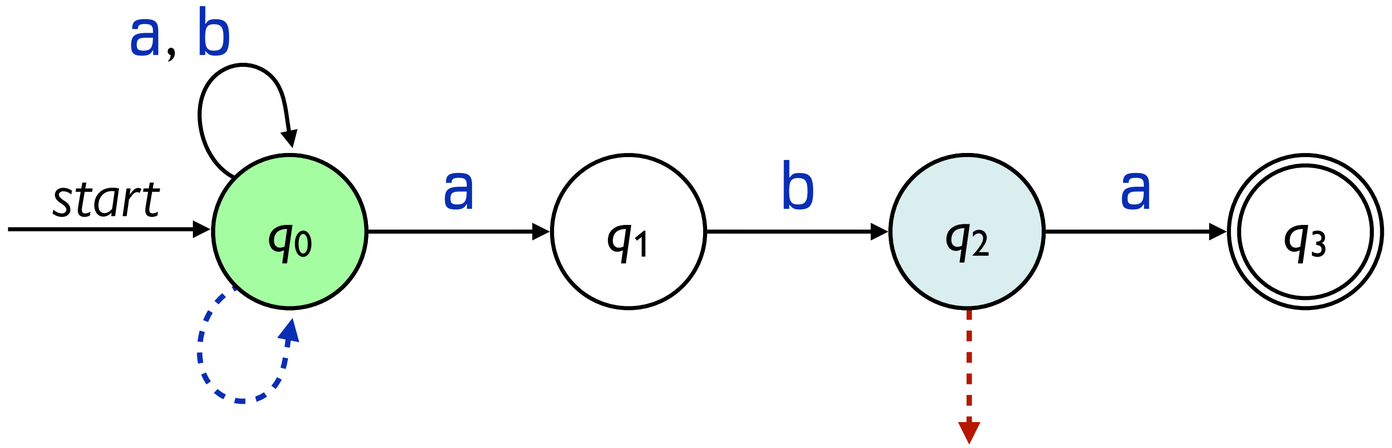
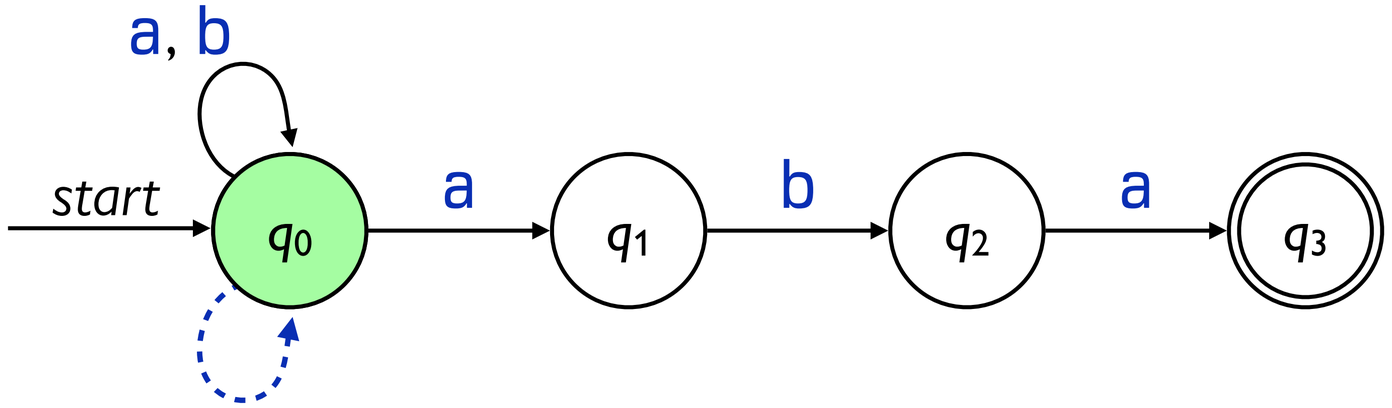
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | |



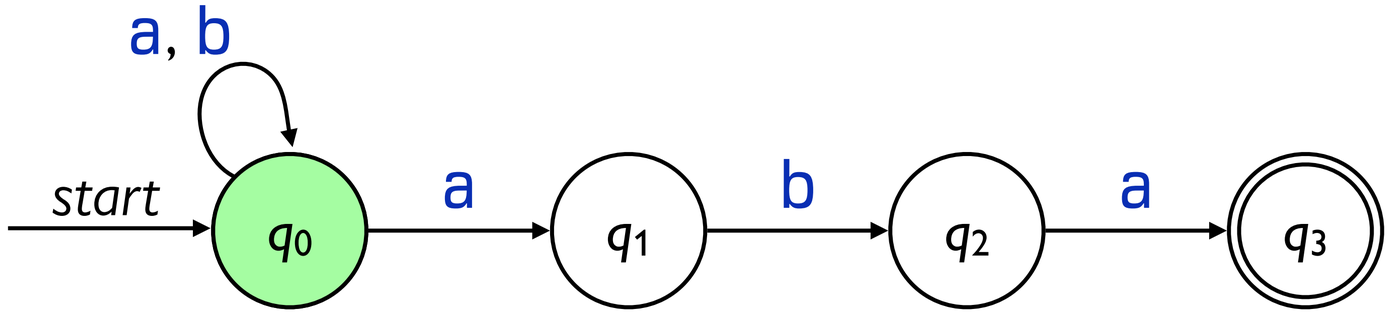

| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | |



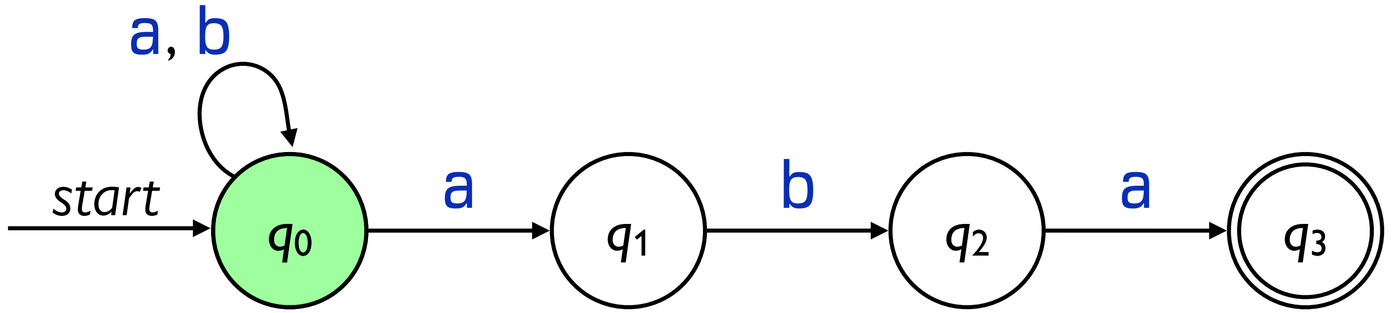

| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | |



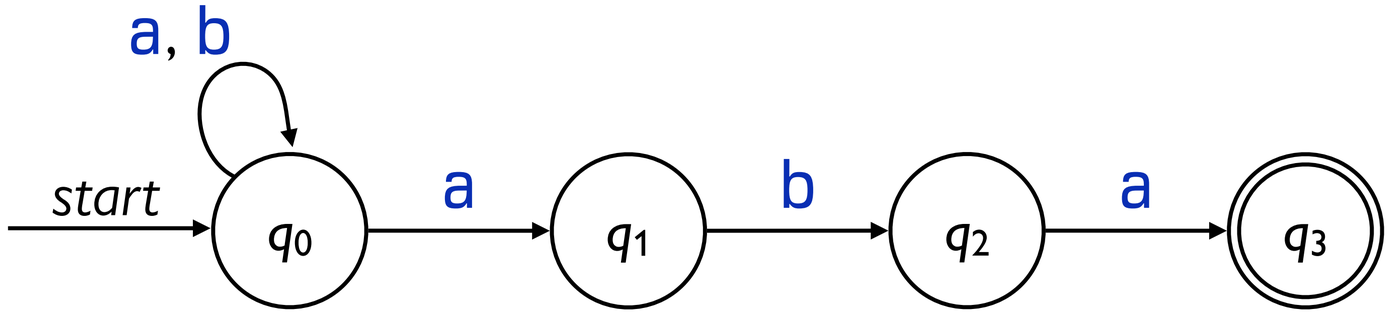

| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | |



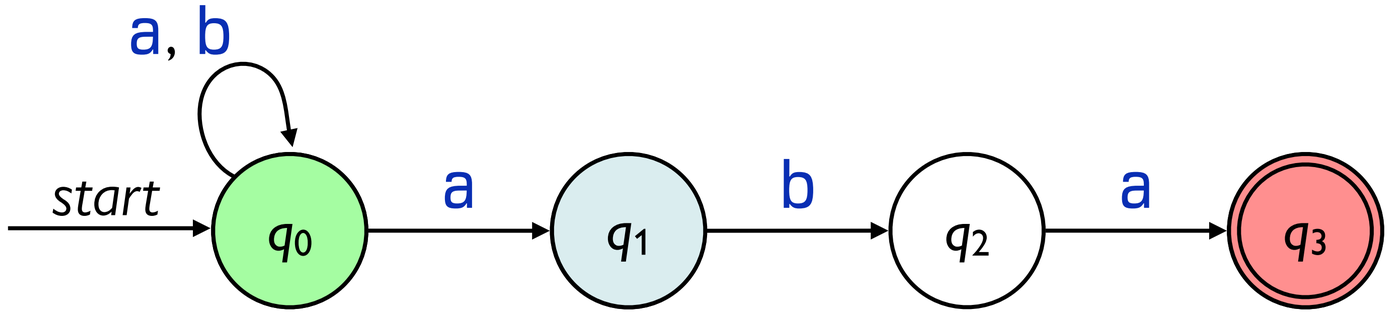

| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | |




| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |




| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |
| | $\{q_0, q_1, q_3\}$ | | |




| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |
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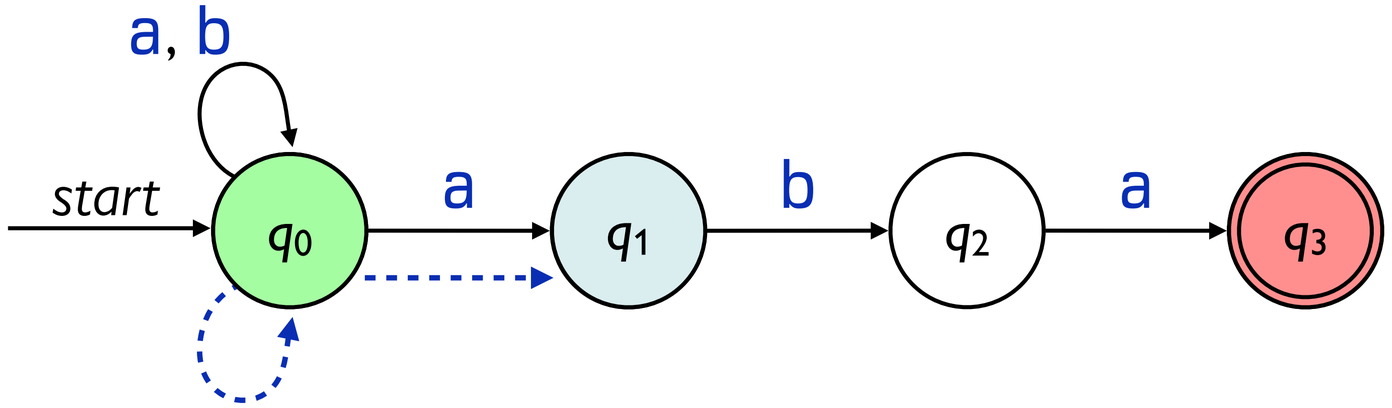


| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
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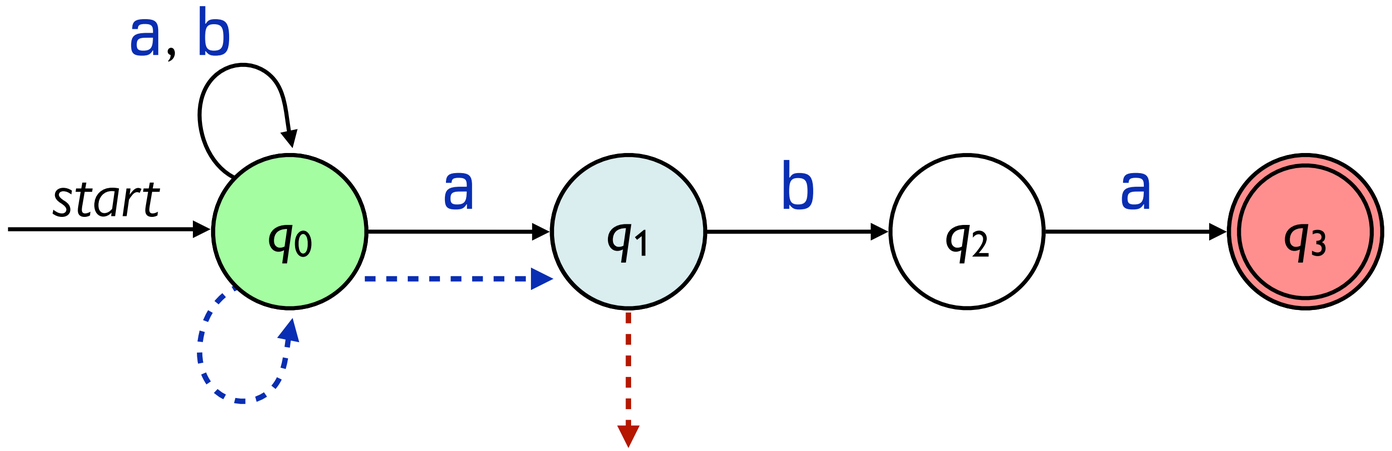


| | State | a | b |
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| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
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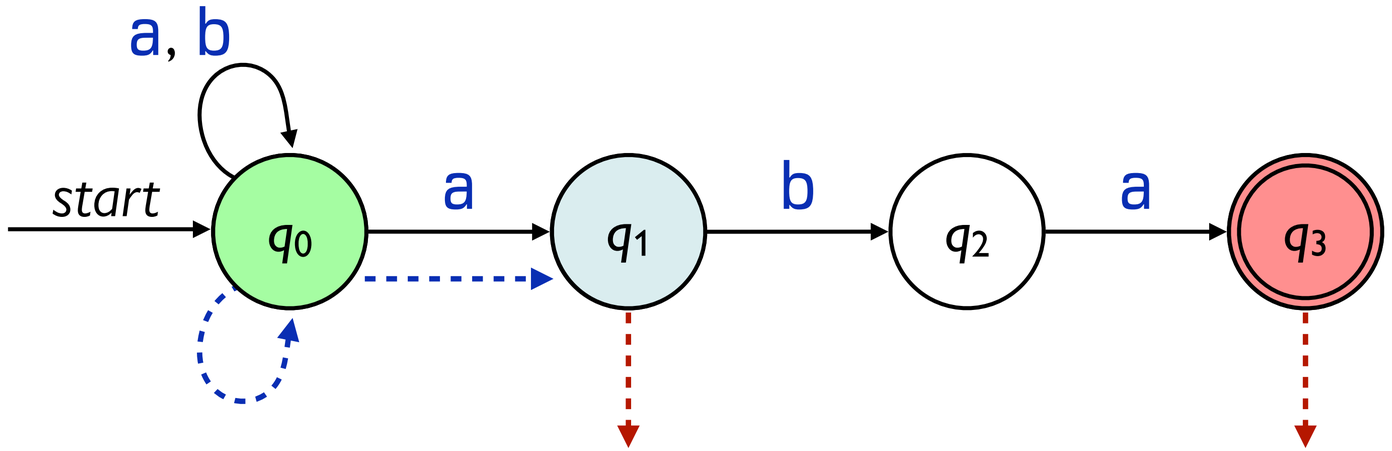


| | State | a | b |
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| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
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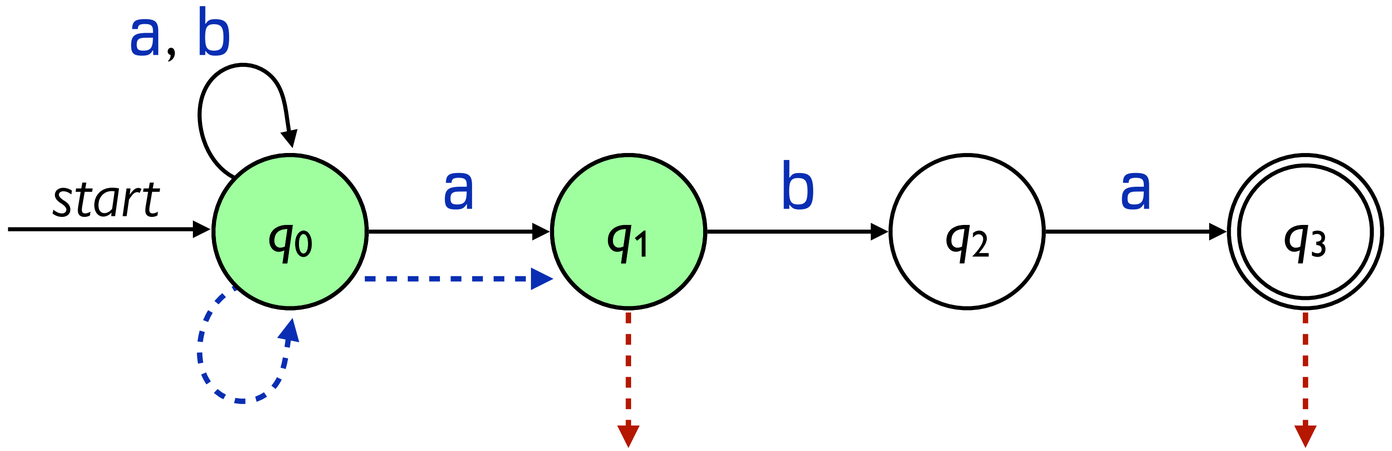


| | State | a | b |
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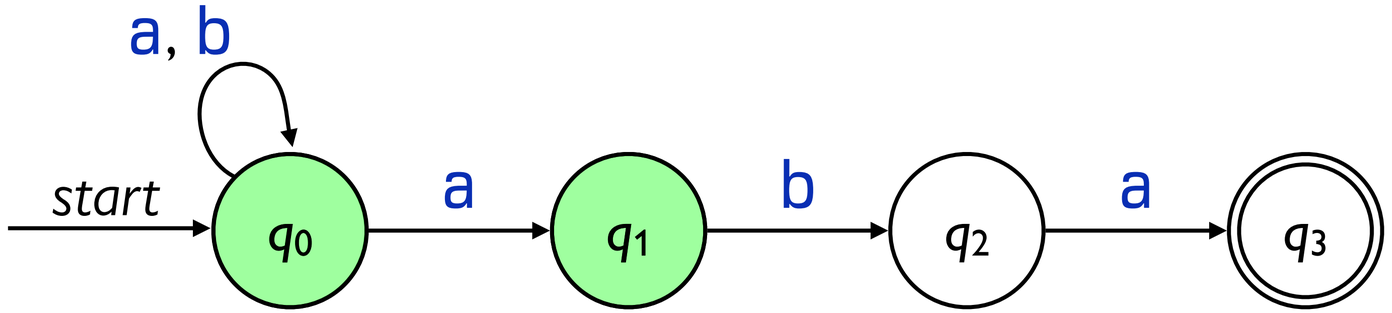


| | State | a | b |
|---|---|---|---|
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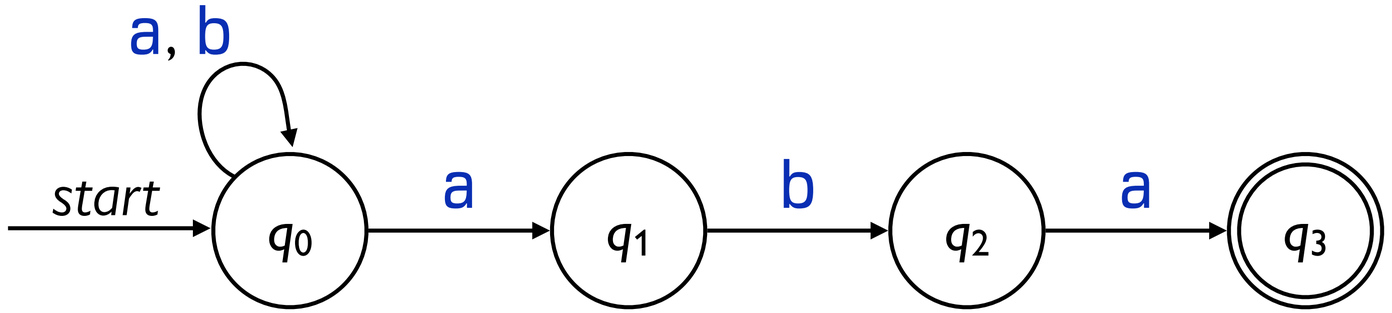


| | State | a | b |
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| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
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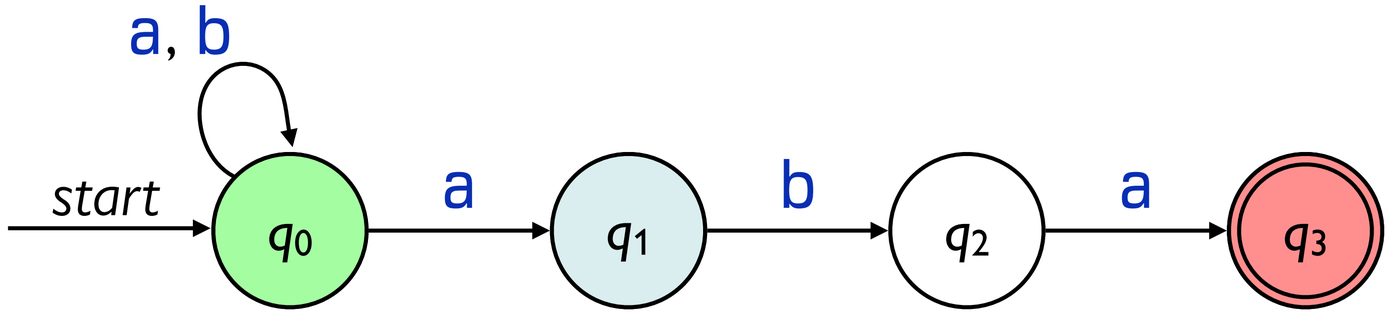


| | State | a | b |
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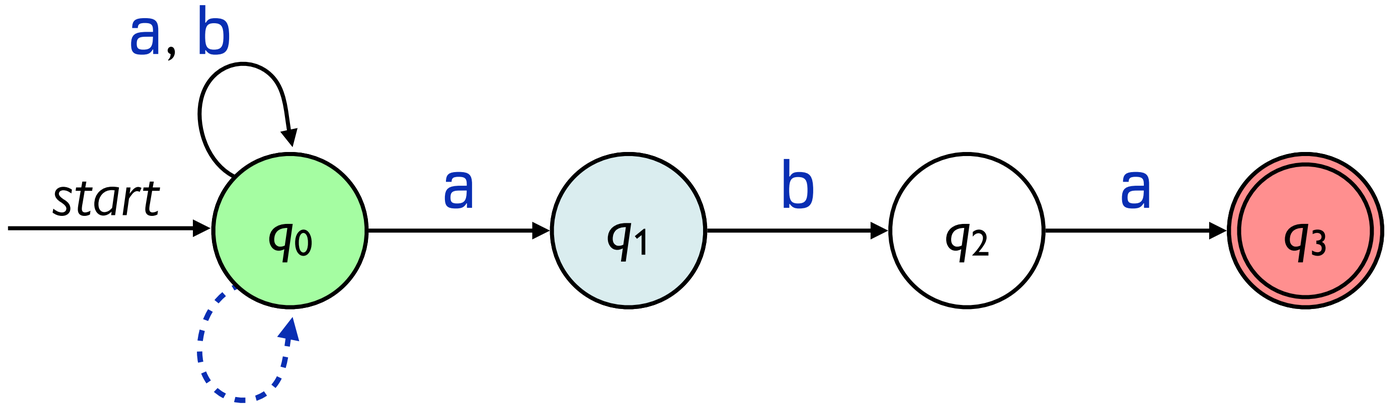


| | State | a | b |
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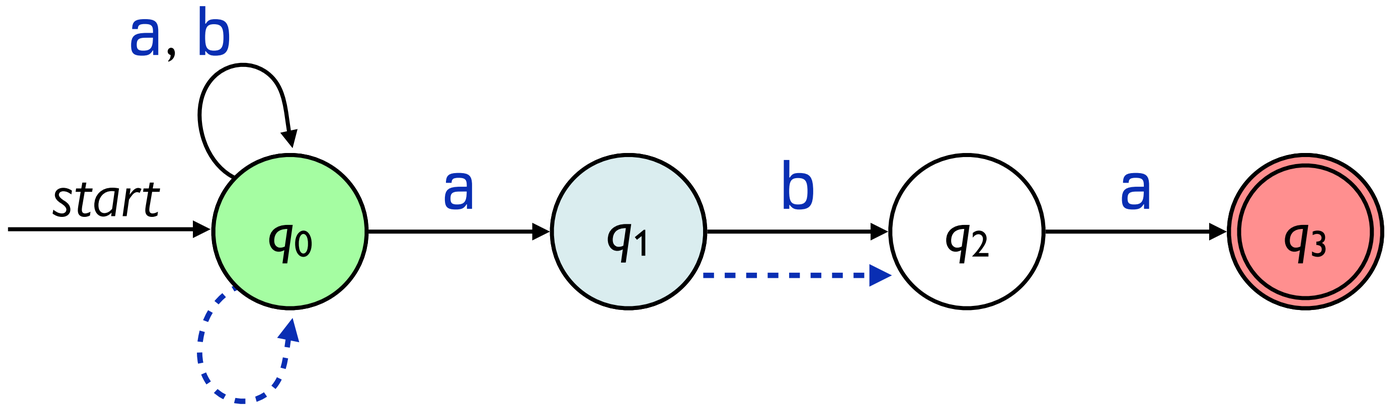


| | State | a | b |
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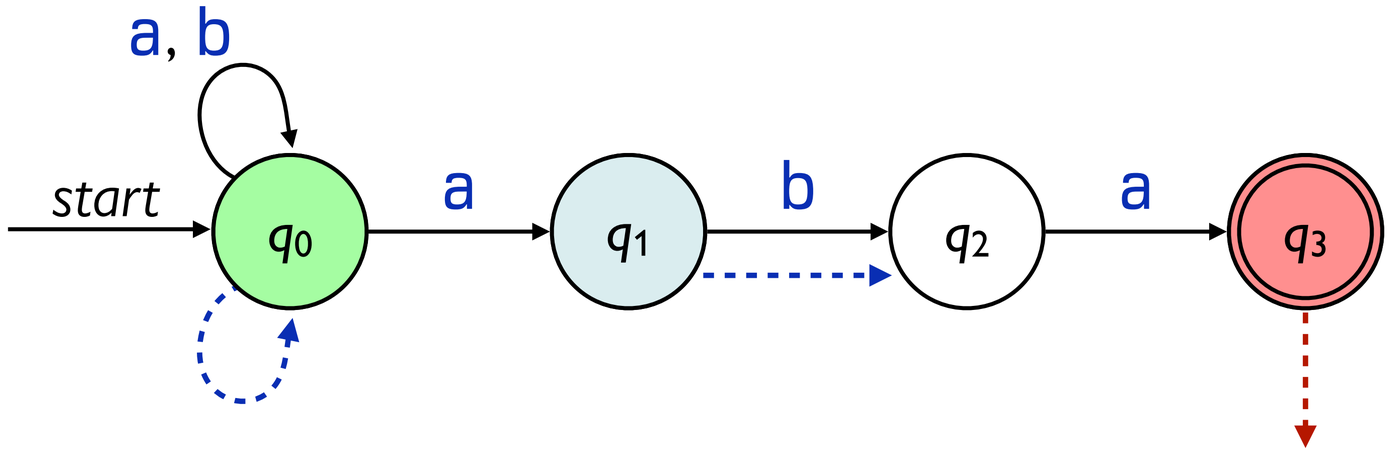


| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
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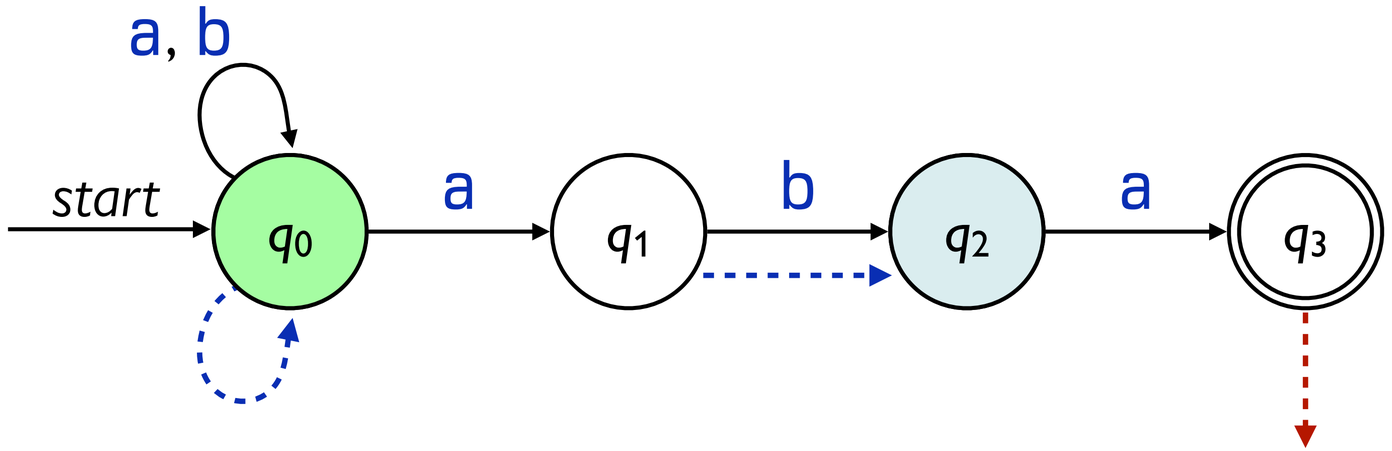


| | State | a | b |
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| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
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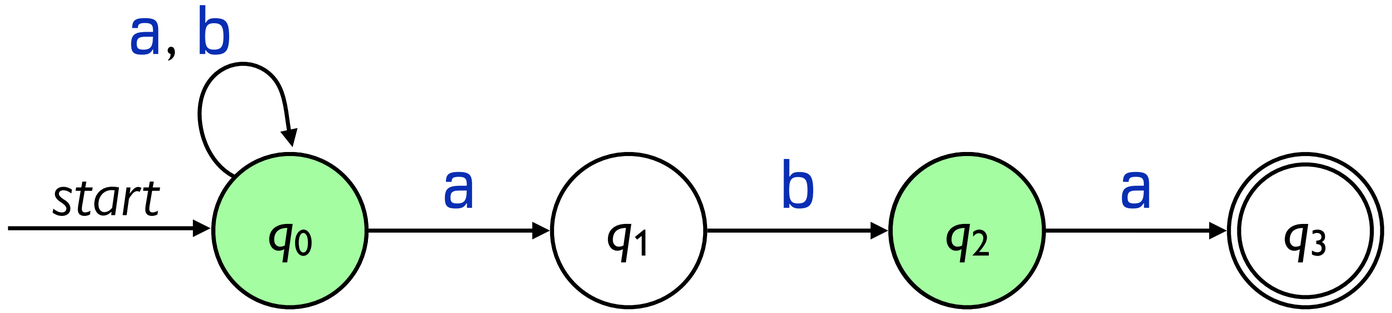


| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
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| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |
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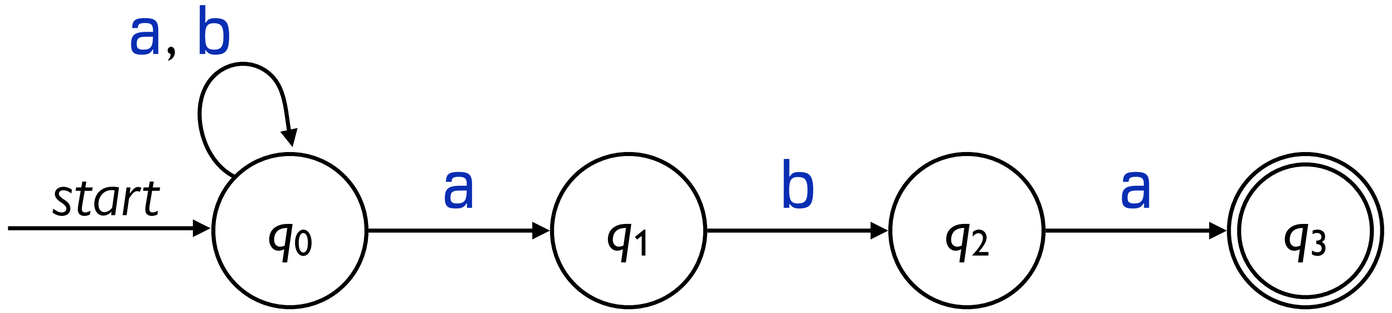
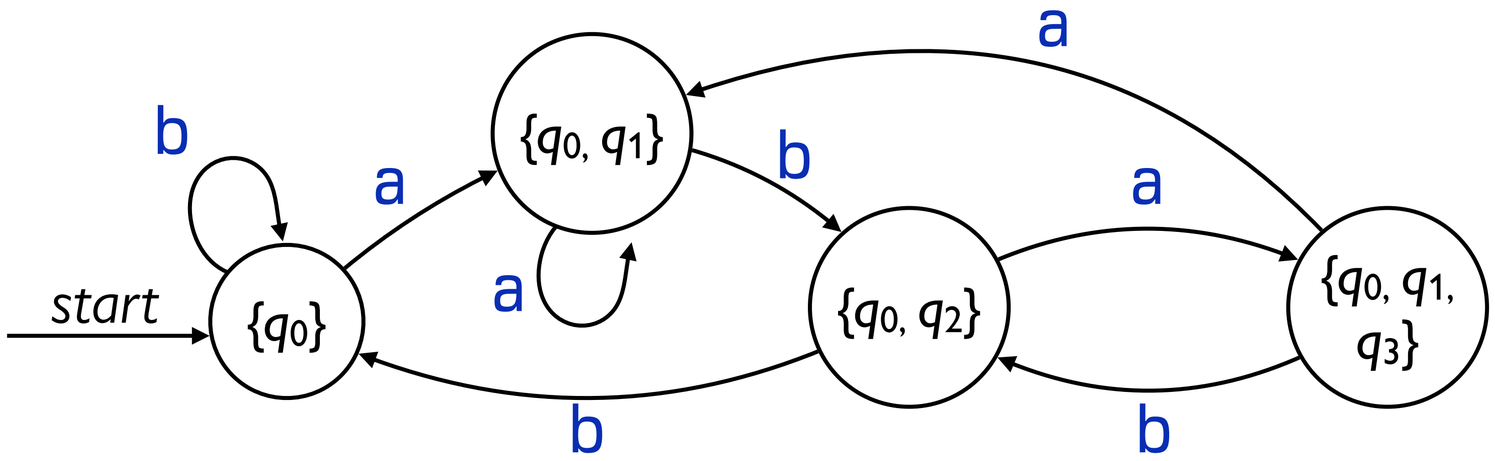
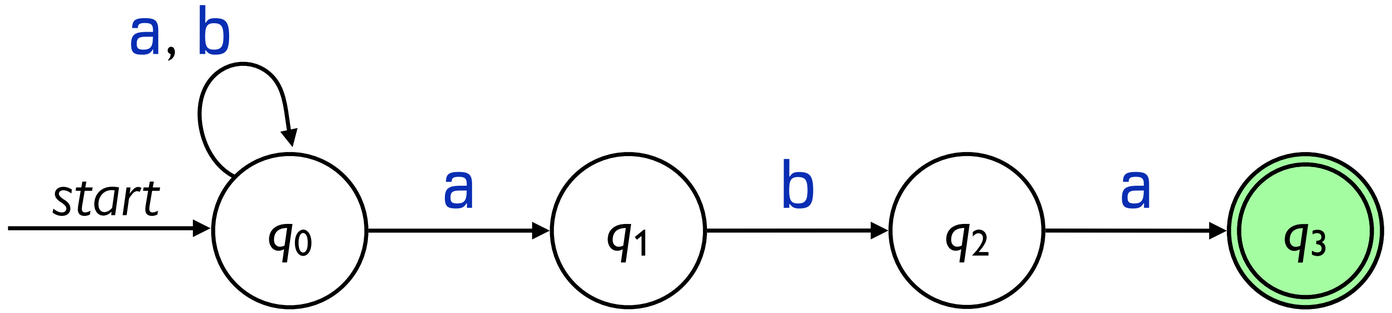
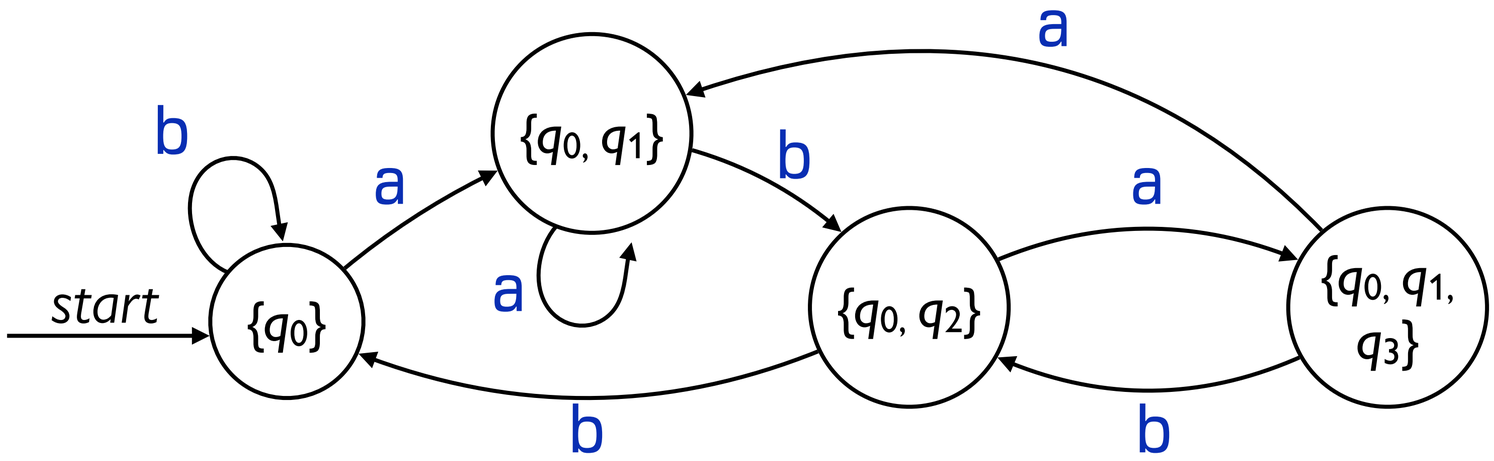


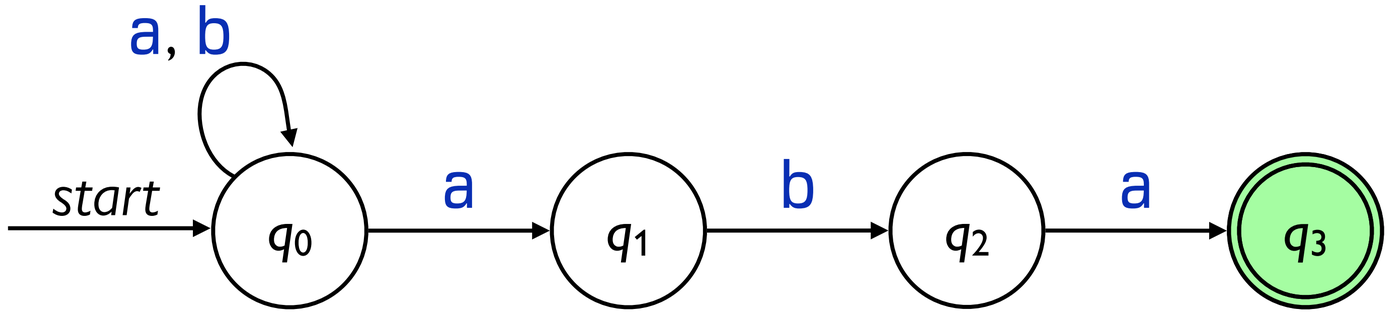

| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |
| | $\{q_0, q_1, q_3\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |



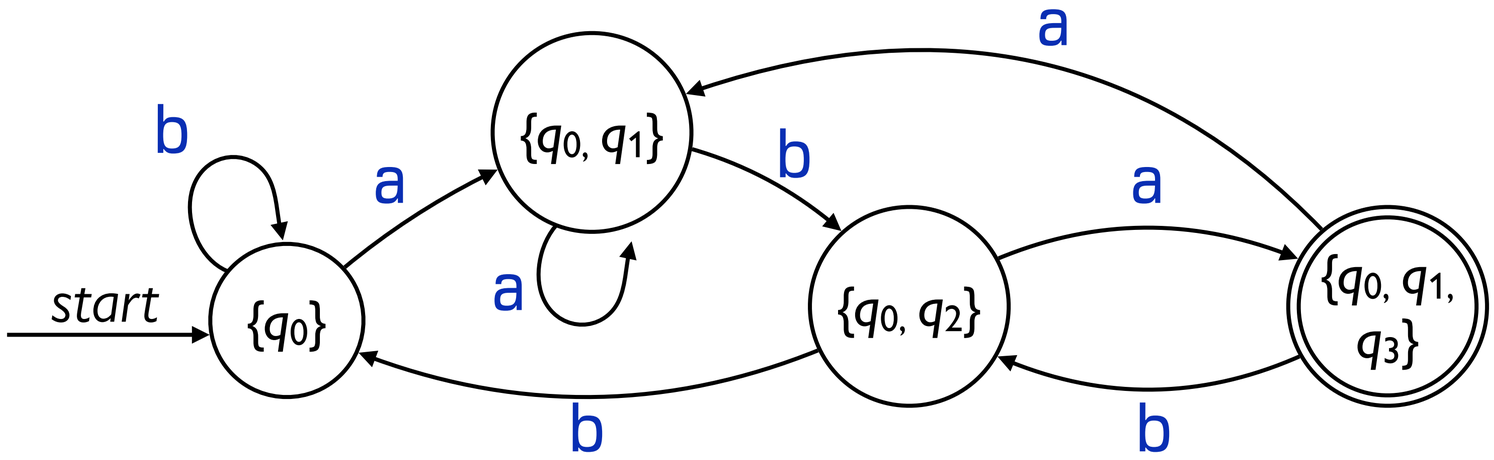


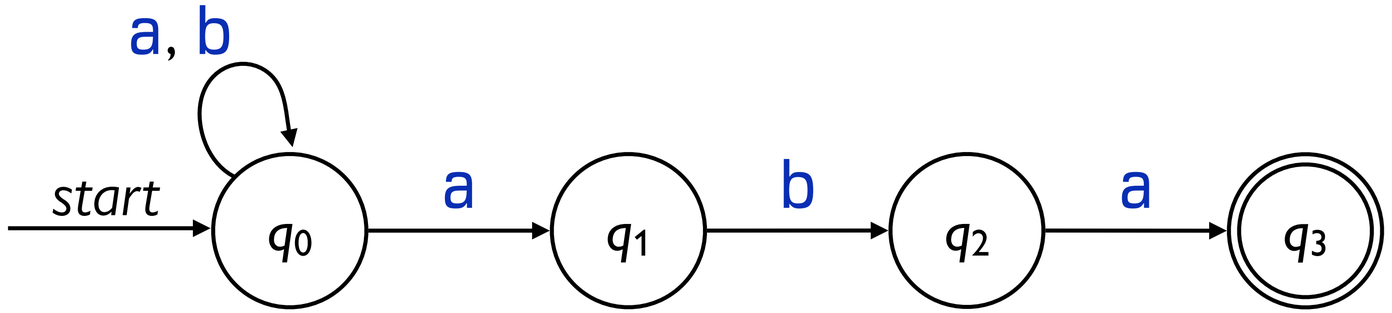

| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |
| | $\{q_0, q_1, q_3\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |

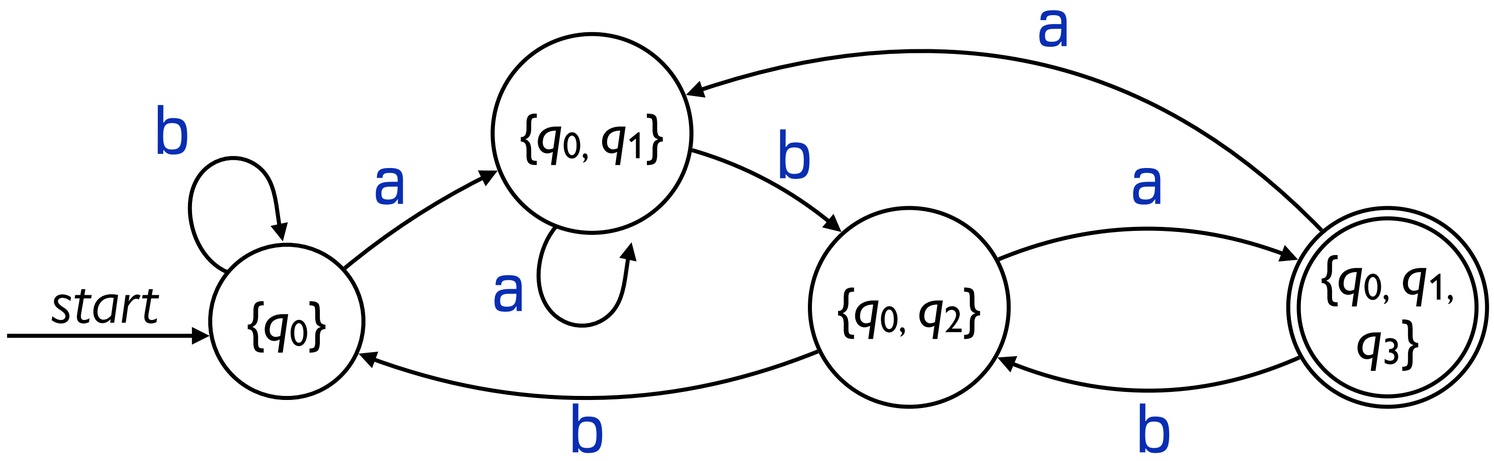


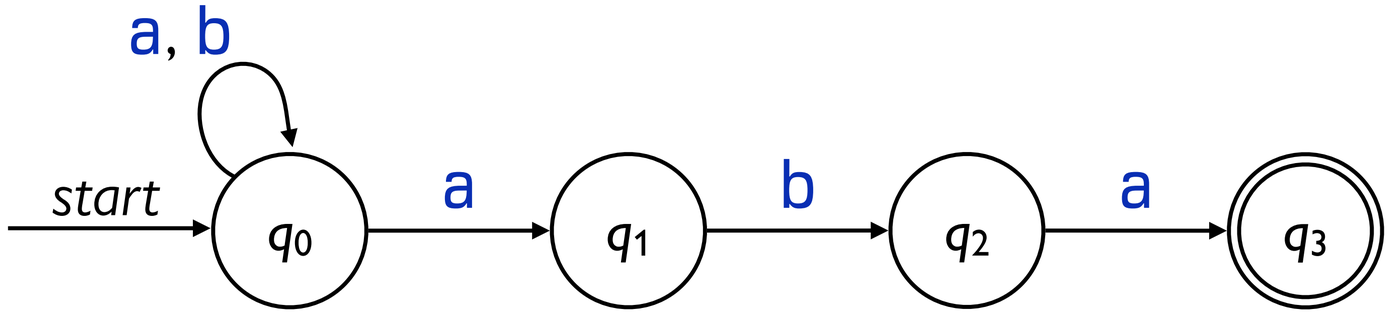



| | State | a | b |
|---|---|---|---|
| → | $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| | $\{q_0, q_2\}$ | $\{q_0, q_1, q_3\}$ | $\{q_0\}$ |
| * | $\{q_0, q_1, q_3\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |

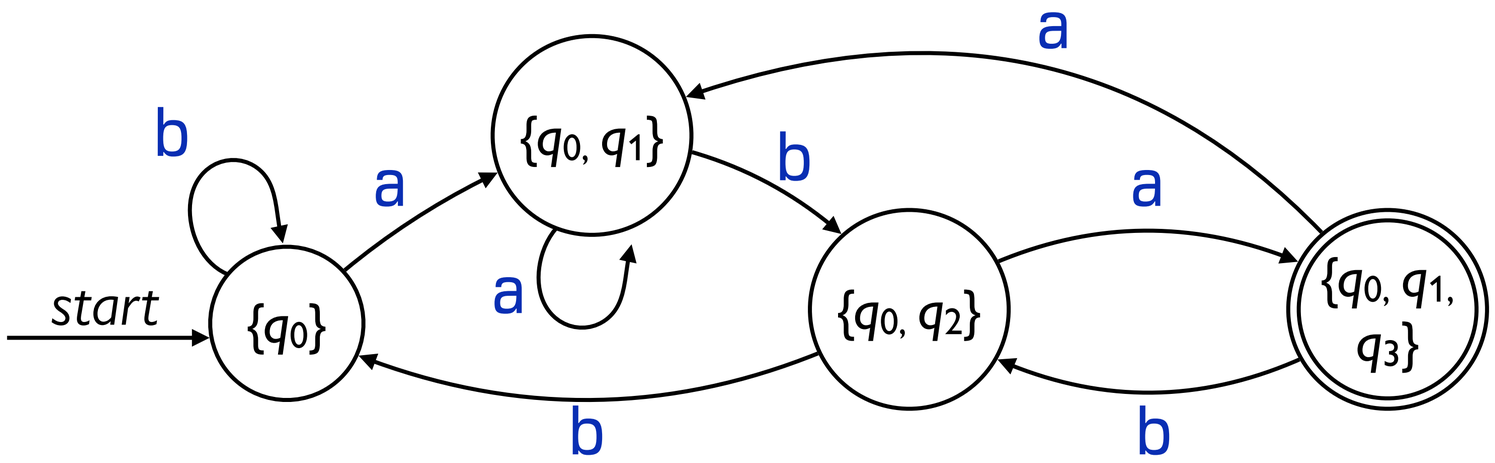


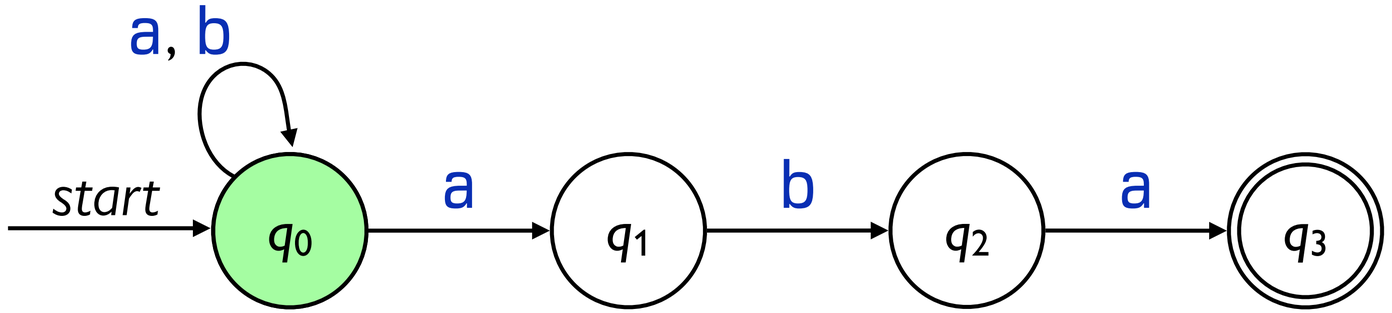

a, b
start
$q_0$
a
$q_1$
b
$q_2$
a
$q_3$


b
a
$\{q_0, q_1\}$
a
b
a
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b

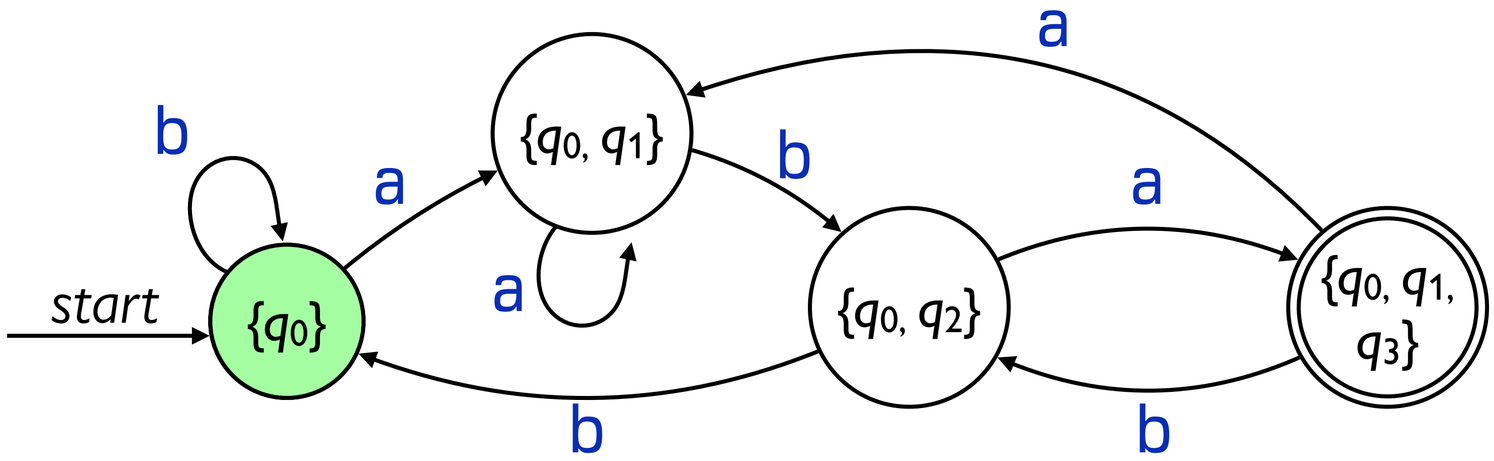


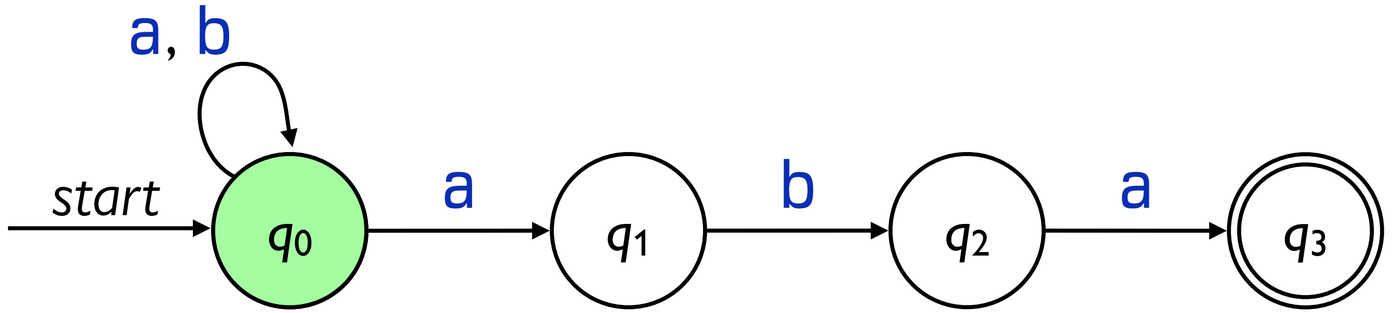

a b a a b a



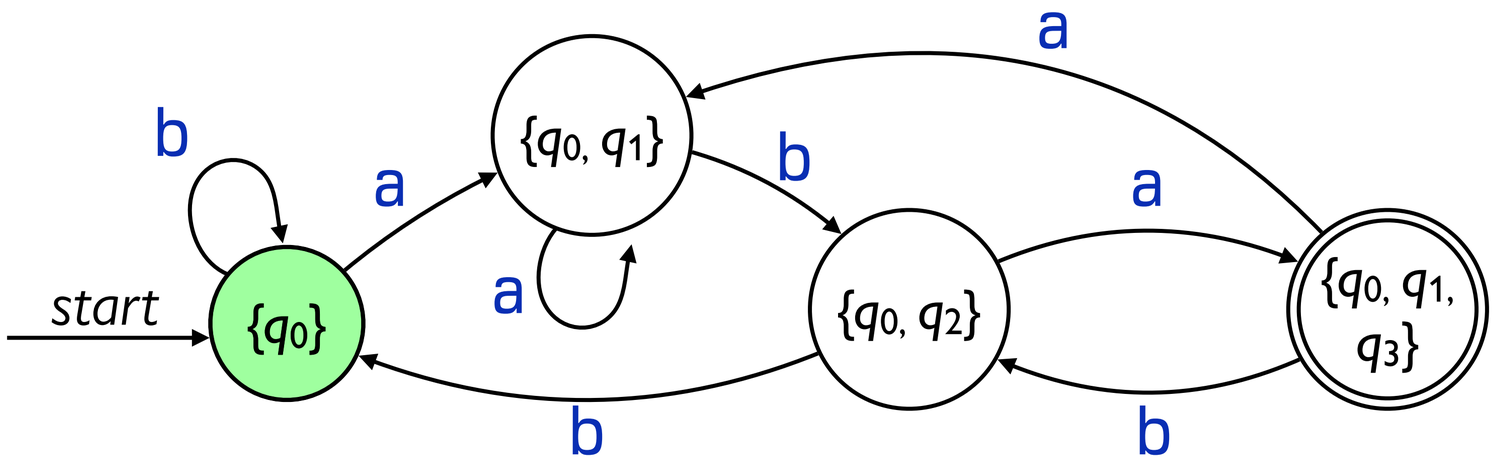



a b a a b a




a, b
start
$q_0$
a
$q_1$
b
$q_2$
a
$q_3$


a b a a b a


b
a
$\{q_0, q_1\}$
a
b
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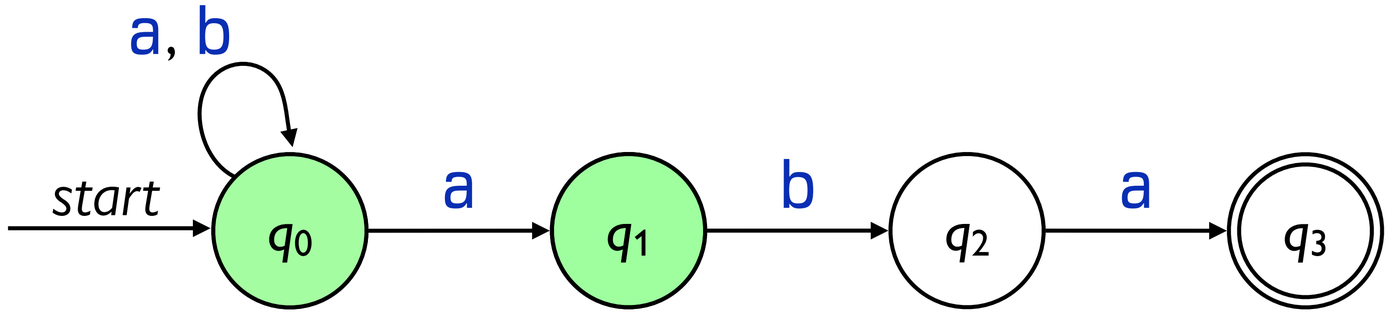
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b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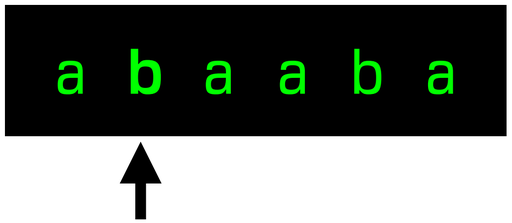
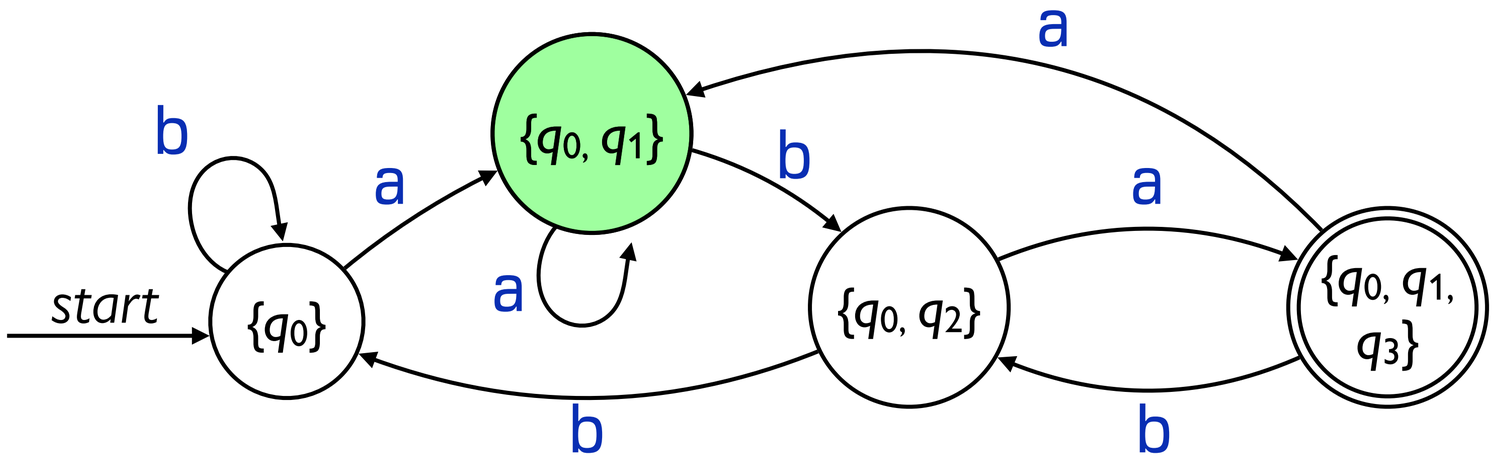

a b a a b a


b
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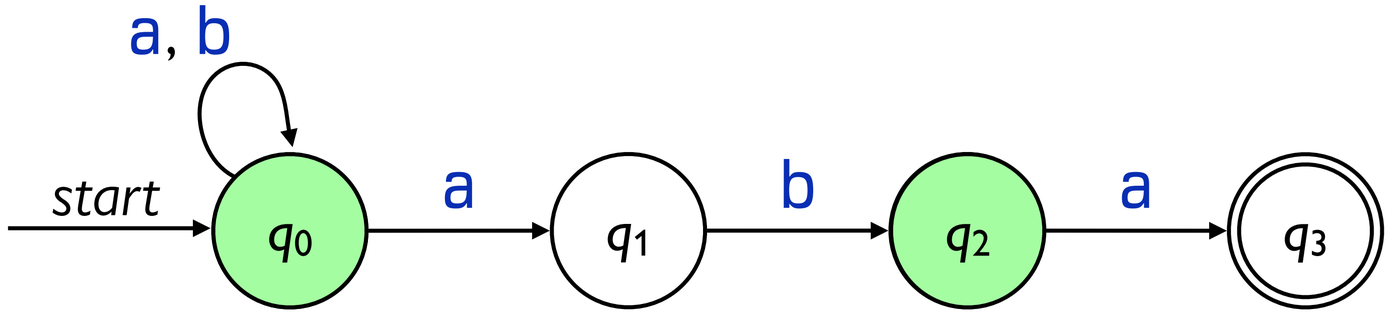
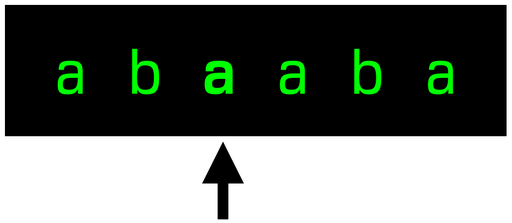
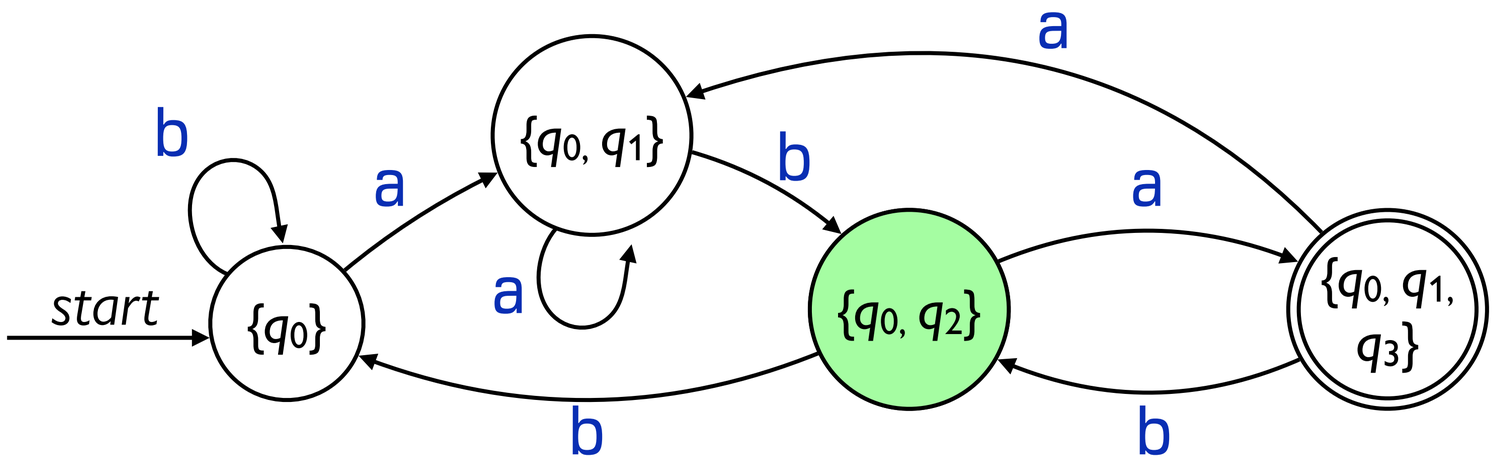
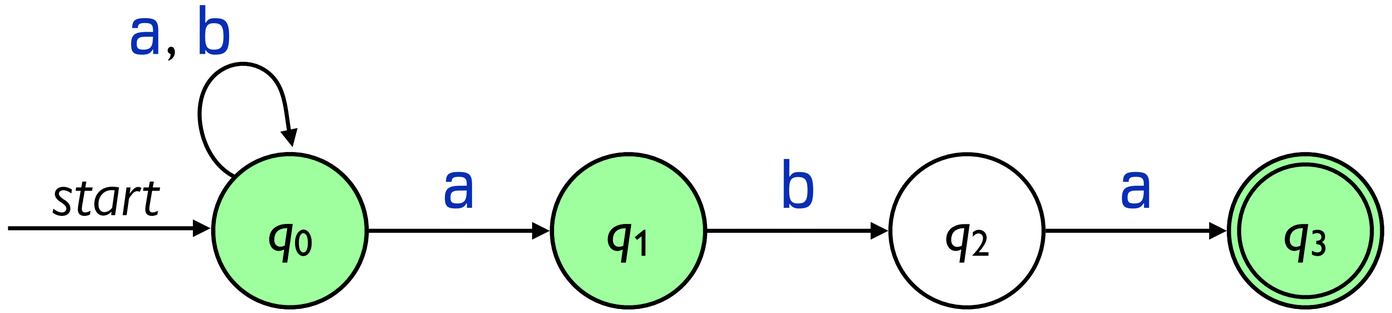
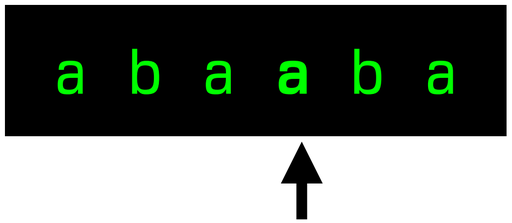
b
a
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b

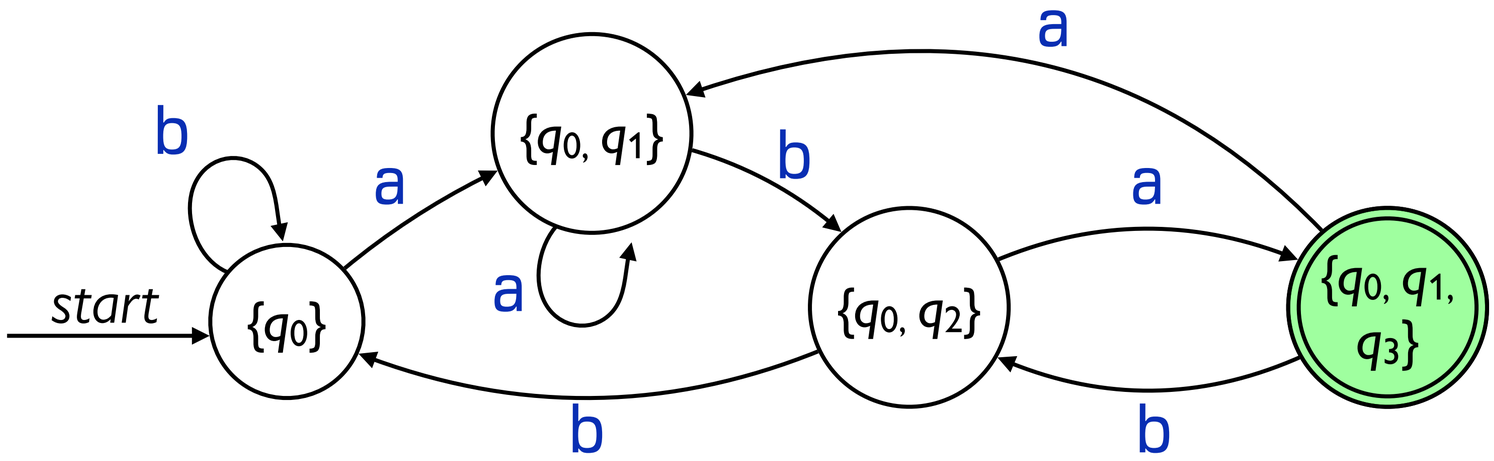

a, b
start
$q_0$
a
$q_1$
b
$q_2$
a
$q_3$

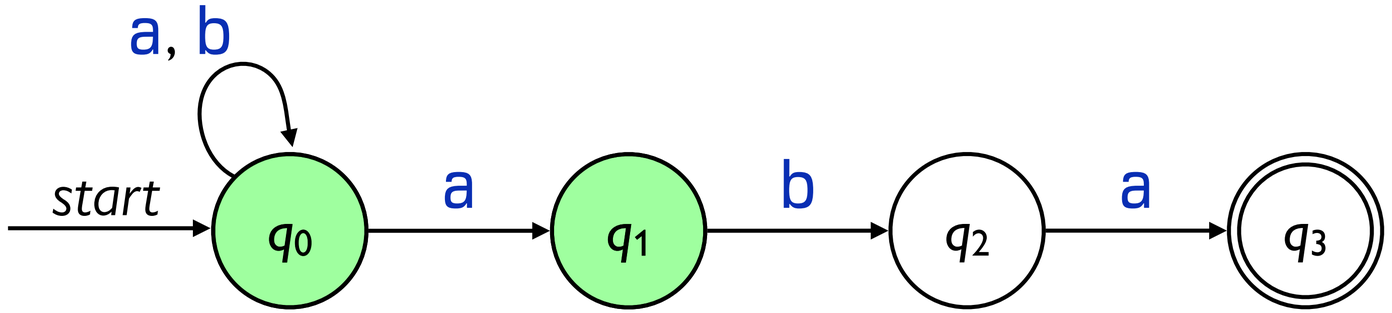

a b a a b a


b
a
$\{q_0, q_1\}$
a
b
a
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b

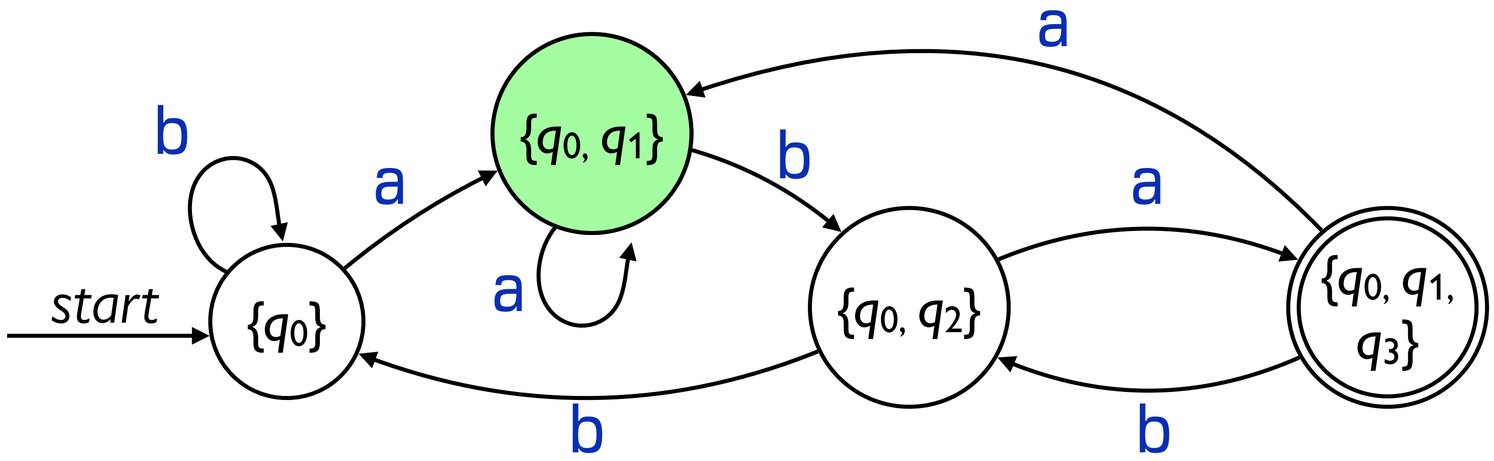

a, b
start
$q_0$
a
$q_1$
b
$q_2$
a
$q_3$

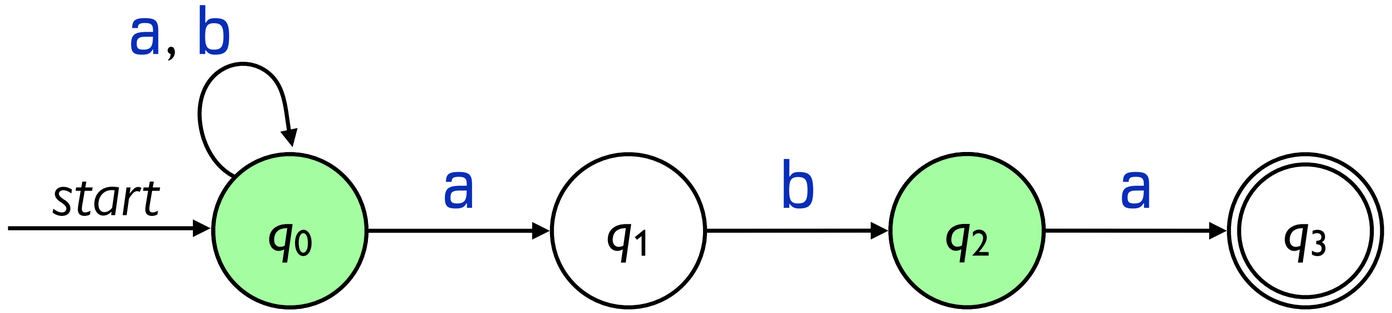

a b a a b a


b
a
$\{q_0, q_1\}$
a
b
a
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b

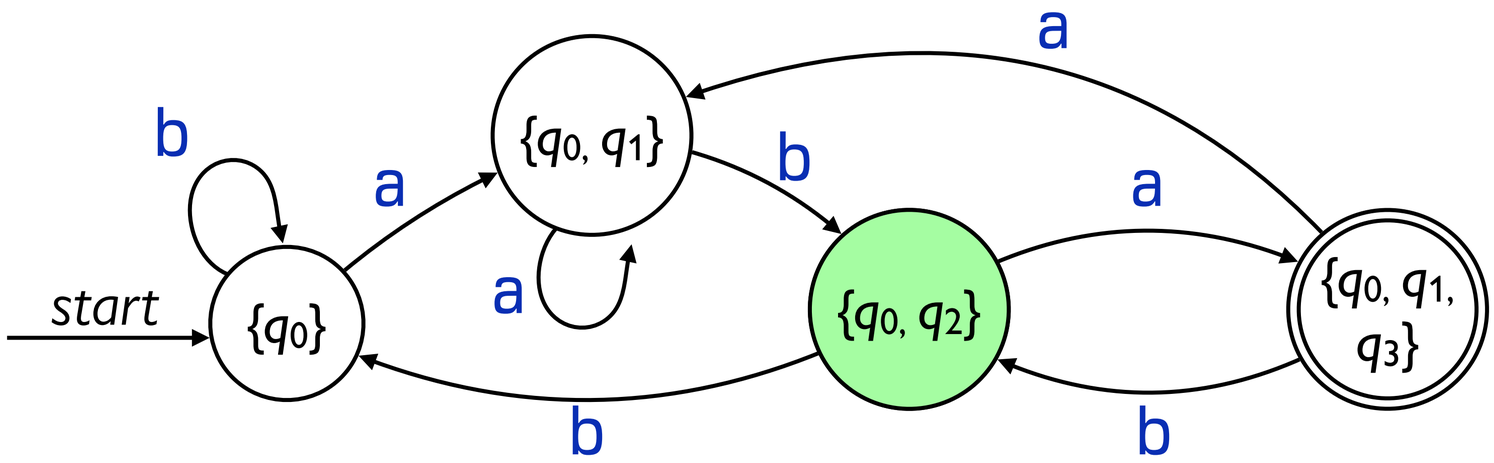

a, b
start
$q_0$
a
$q_1$
b
$q_2$
a
$q_3$

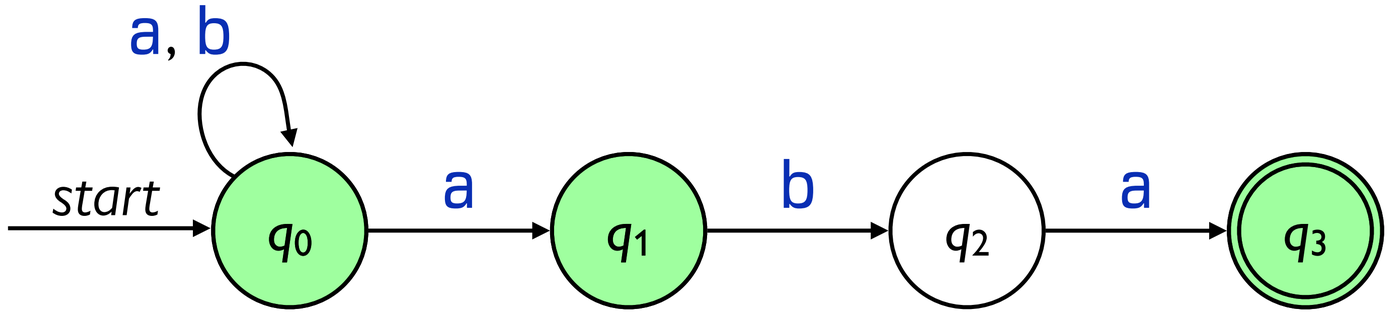

a b a a b a


b
a
$\{q_0, q_1\}$
a
b
a
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b

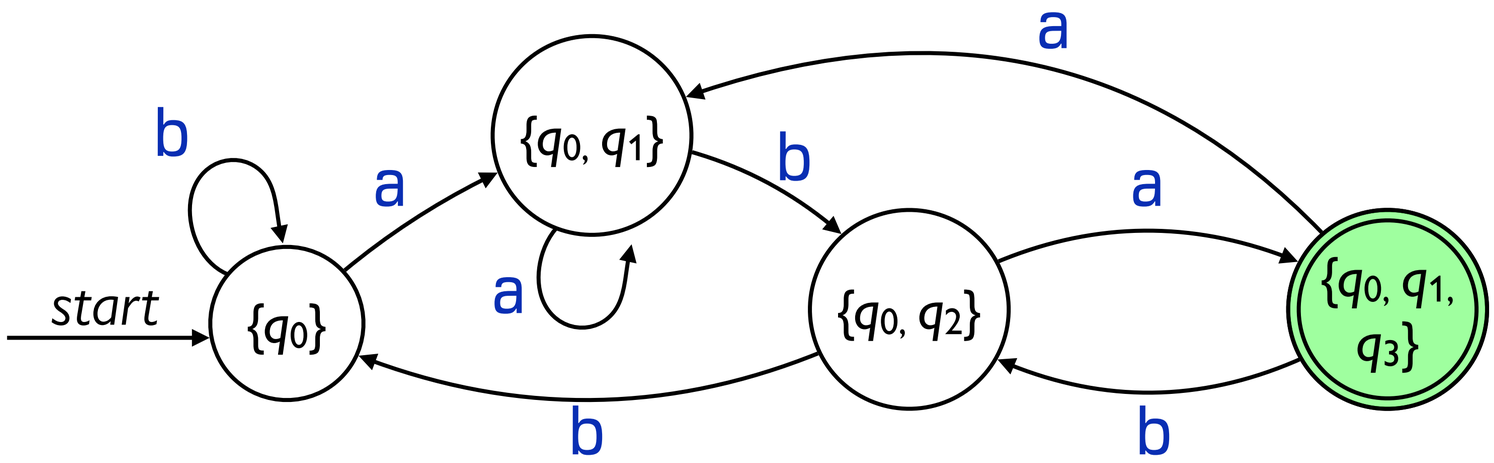

a, b
start
$q_0$
a
$q_1$
b
$q_2$
a
$q_3$


a b a a b a


b
a
$\{q_0, q_1\}$
a
b
a
start
$\{q_0\}$
a
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$
b
b




a b a a b a



This method of transforming an NFA for a language *L* into a DFA for *L* is called the ***subset construction***.

*Introduced by Rabin & Scott, 1959*

Each state in the DFA corresponds to a ***set*** of states in the NFA.

The start state in the DFA corresponds to the start state of the NFA.

The accept states in the DFA correspond to the sets of states that would be considered to accept in the NFA when using the massive parallelism intuition.

If a state *q* in the DFA corresponds to a set of states *S* in the NFA, then the transition from state *q* on a character *a* is found as follows:

Let *S′* be the set of states in the NFA that can be reached by following a transition labeled *a* from any of the states in *S*.

The state *q* in the DFA transitions on *a* to a DFA state corresponding to the set of states *S′*.

In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.

Useful fact: $|\wp(S)| = 2^{|S|}$ for any finite set $S$.

In the worst case, the construction can result in a DFA that is *exponentially larger* than the original NFA.

A language $L$ is called a *regular language* if there exists a DFA $D$ such that $L(D) = L$.

**THEOREM** Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

**PROOF** Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA recognizing some language $A$.

We can construct a DFA $D = (Q', \Sigma, \delta', q_0', F')$ that recognizes $A$:

$Q' = \wp(Q)$

$q_0' = \{q_0\}$

$F' = \{R \in Q' \mid R \text{ contains an accept state of } N\}$

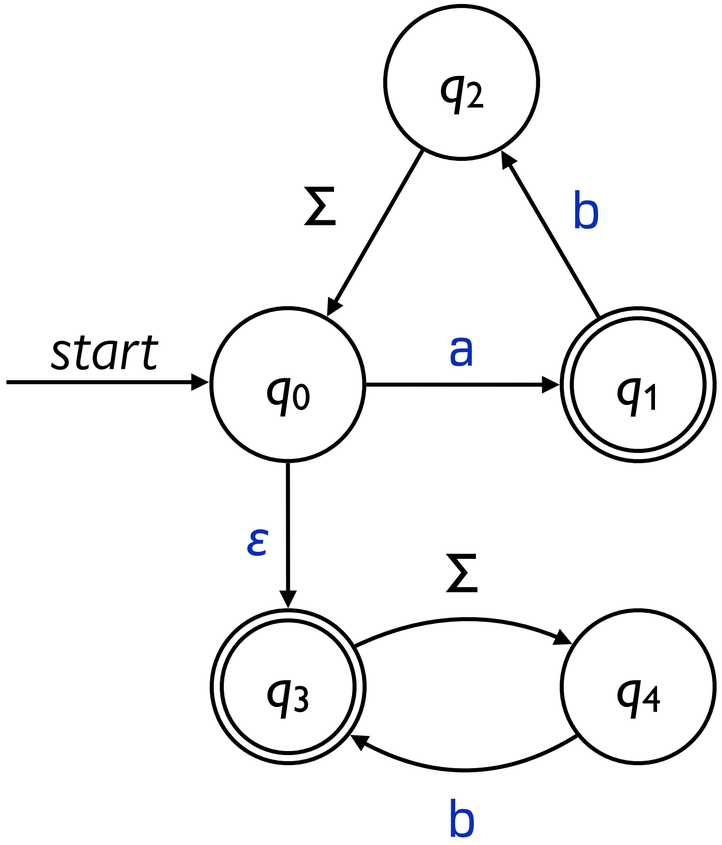
For $R \in Q'$ and $a \in \Sigma$, we define $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$

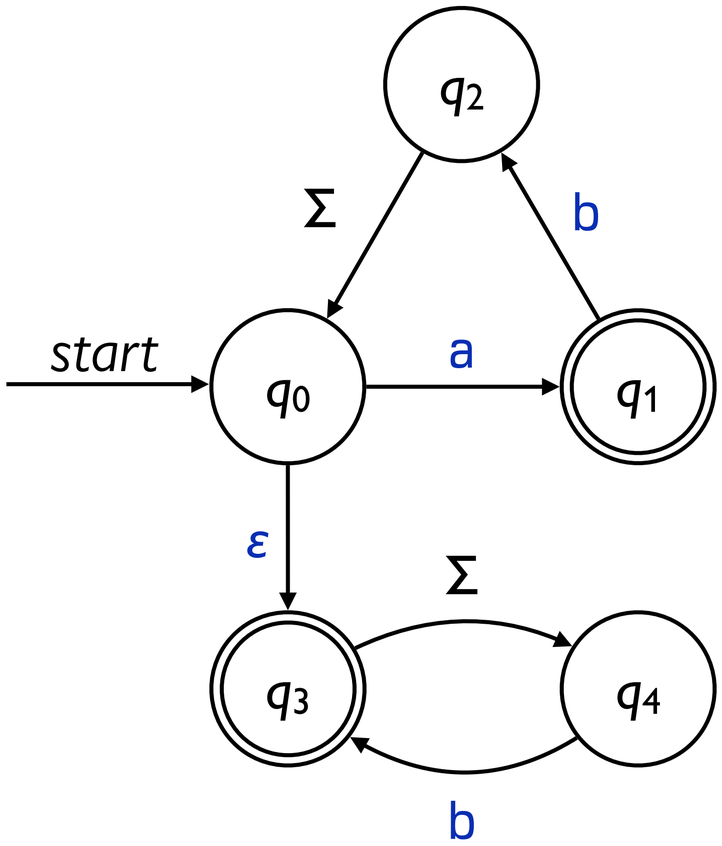
*As in Sipser, we're using R both as a **state** of D and as a **set of states** of N.*

Every state $R$ of $D$ is a set of states of $N$.

When $D$ is in state $R$ and reads a symbol $a$, it tracks where $N$ would go on $a$ each state in $R$.

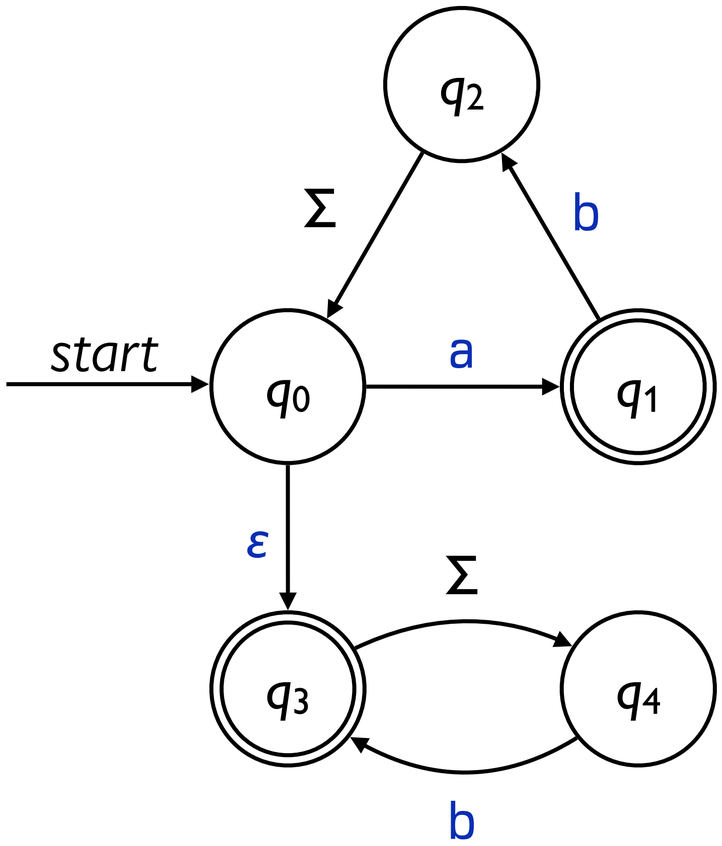
# Wait… What about NFAs with ε-transitions?


$q_2$
Σ
b
start
$q_0$
a
$q_1$
ε
Σ
$q_3$
$q_4$
b



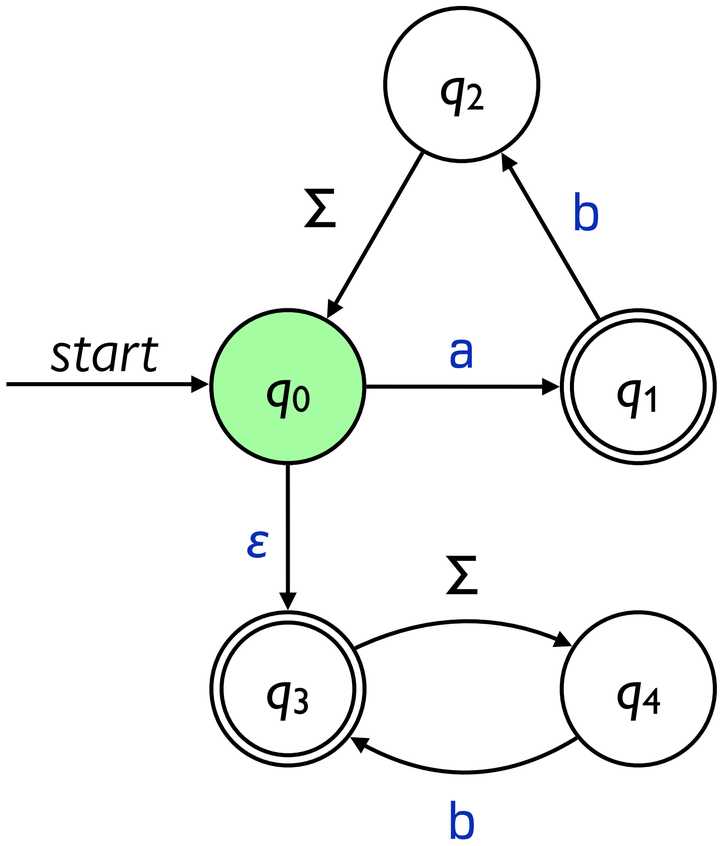

| State | a | b |
|---|---|---|

*It'll be easier for us to represent the equivalent DFA as a table rather than a (big, messy) diagram.*



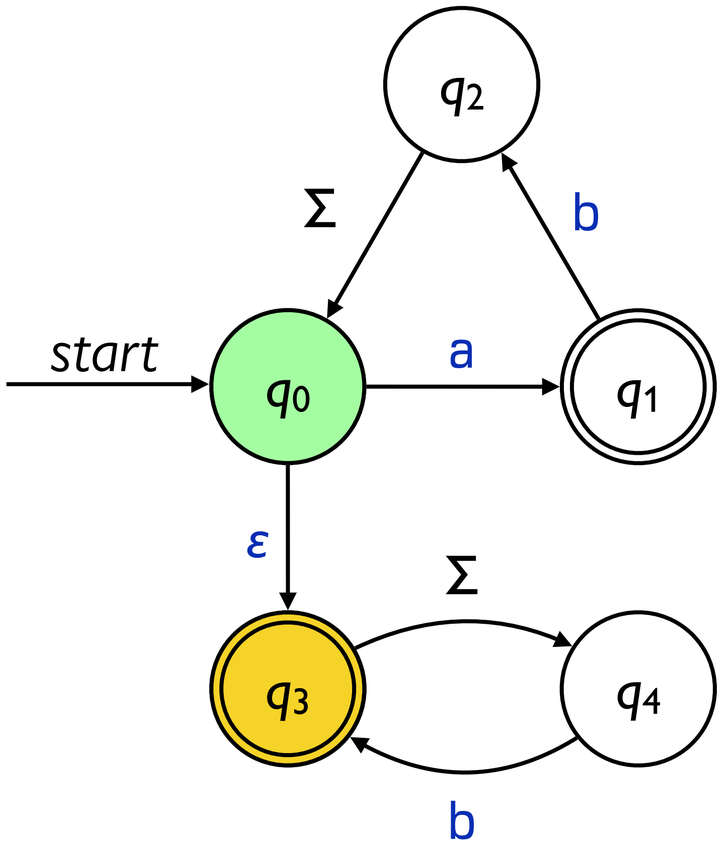

| State | a | b |
|---|---|---|

*Let's start off by thinking what the start state of our DFA is going to be.*



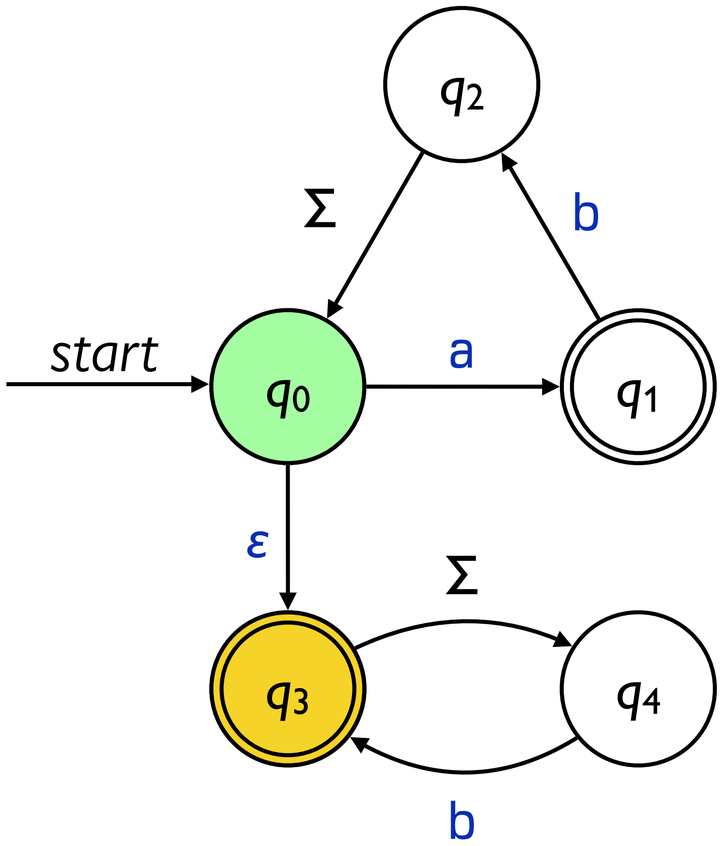

| State | a | b |
|---|---|---|

*Let's start off by thinking what the start state of our DFA is going to be.*



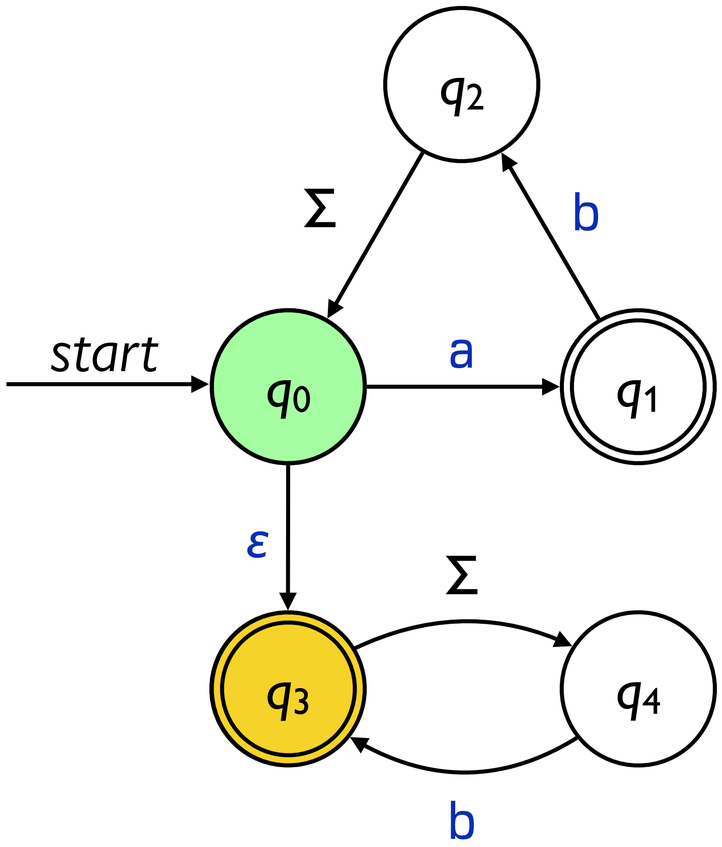

| State | a | b |
|---|---|---|

*Let's start off by thinking what the start state of our DFA is going to be.*

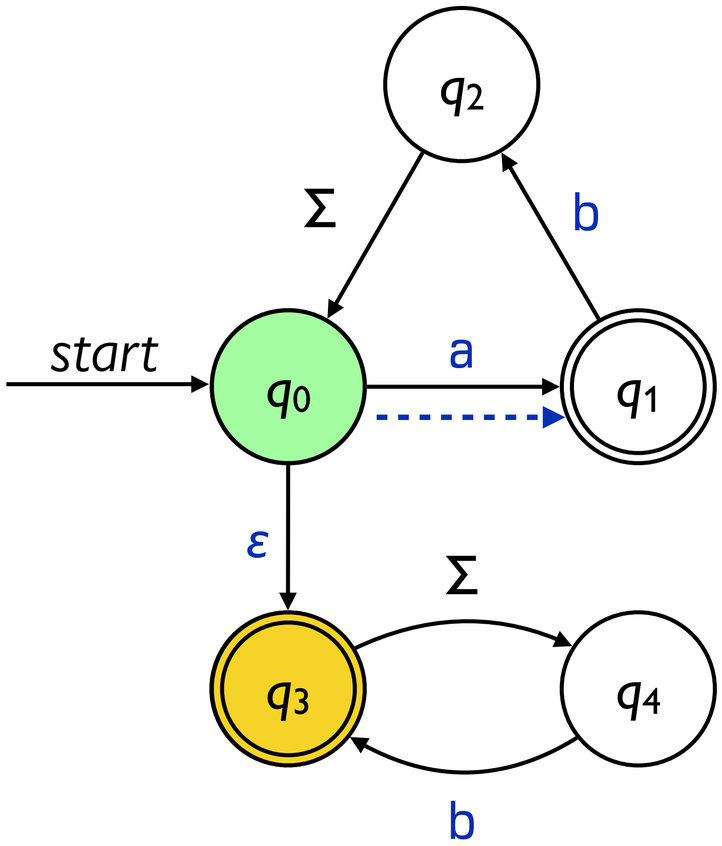



| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | | |

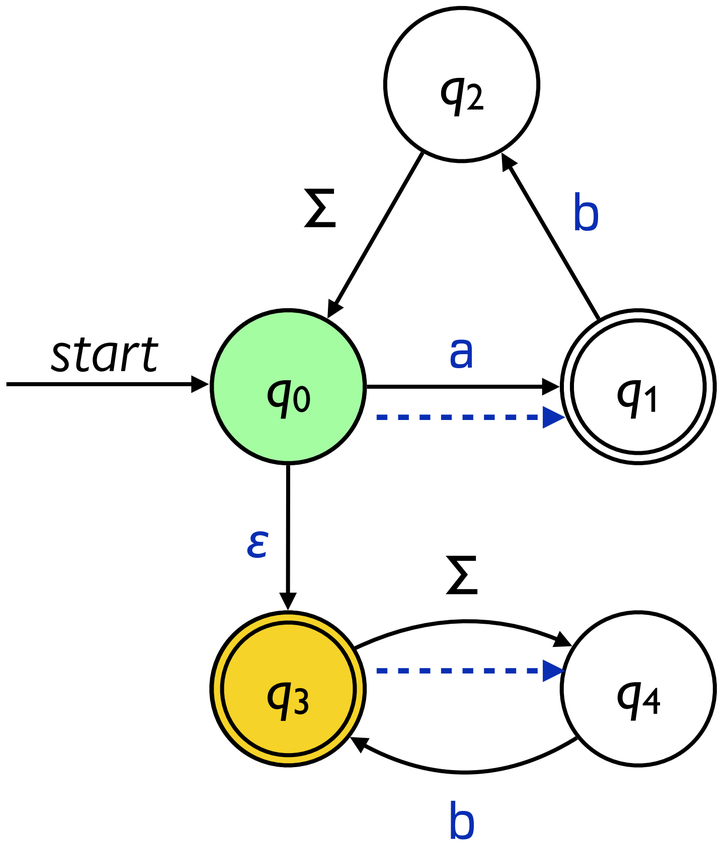
*The start state of the NFA includes the start state $q_0$ of the DFA and $q_3$ since you can get to $q_3$ from $q_0$ by an ε-transition.*



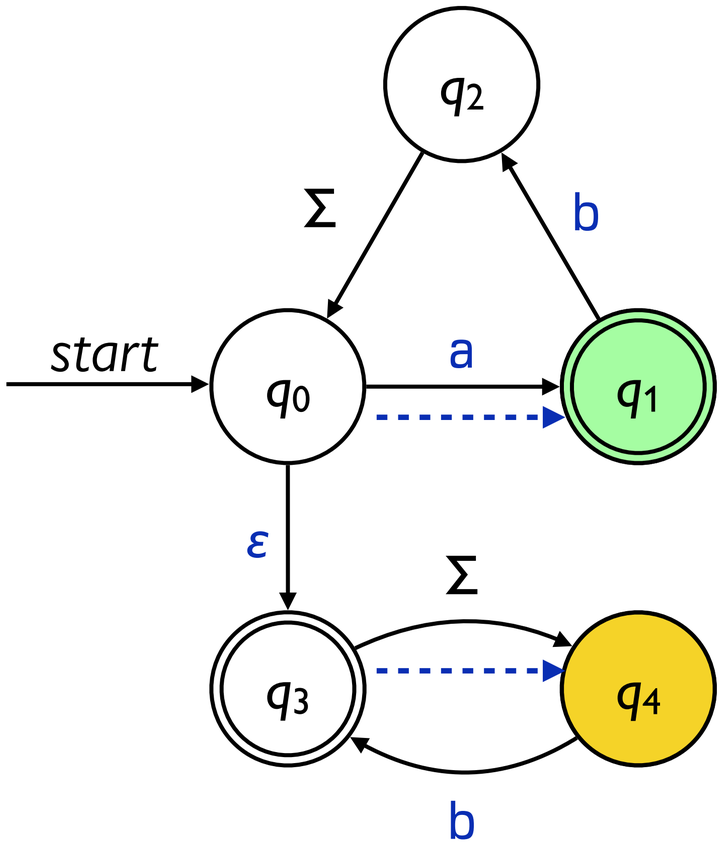

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | | |



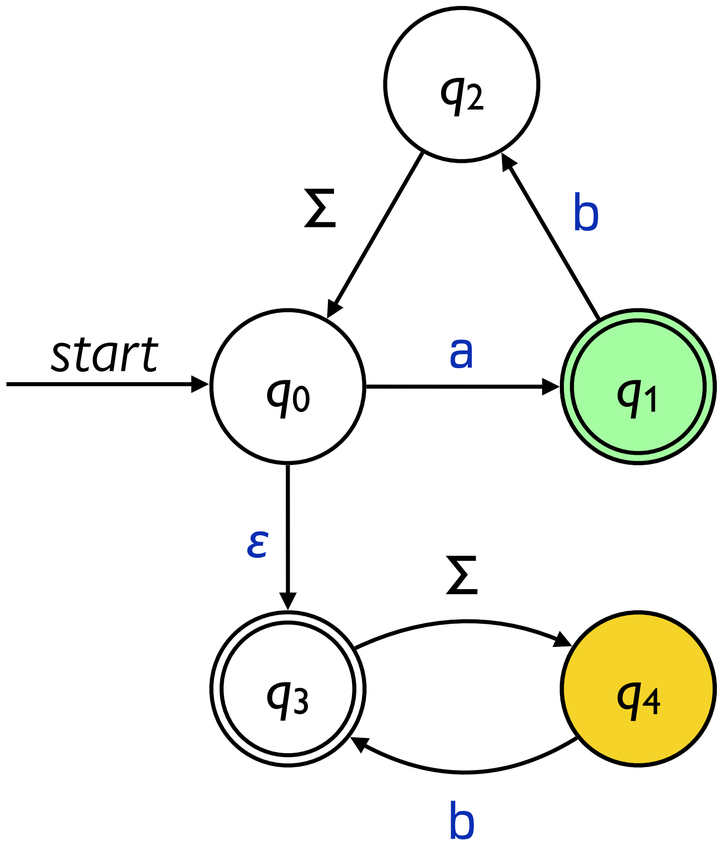

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | | |



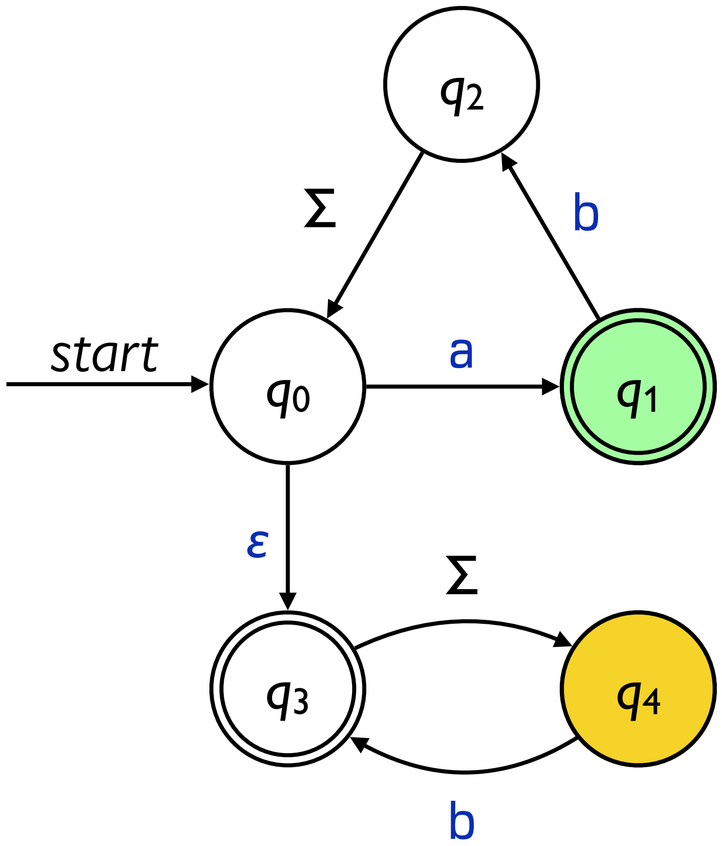

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | | |



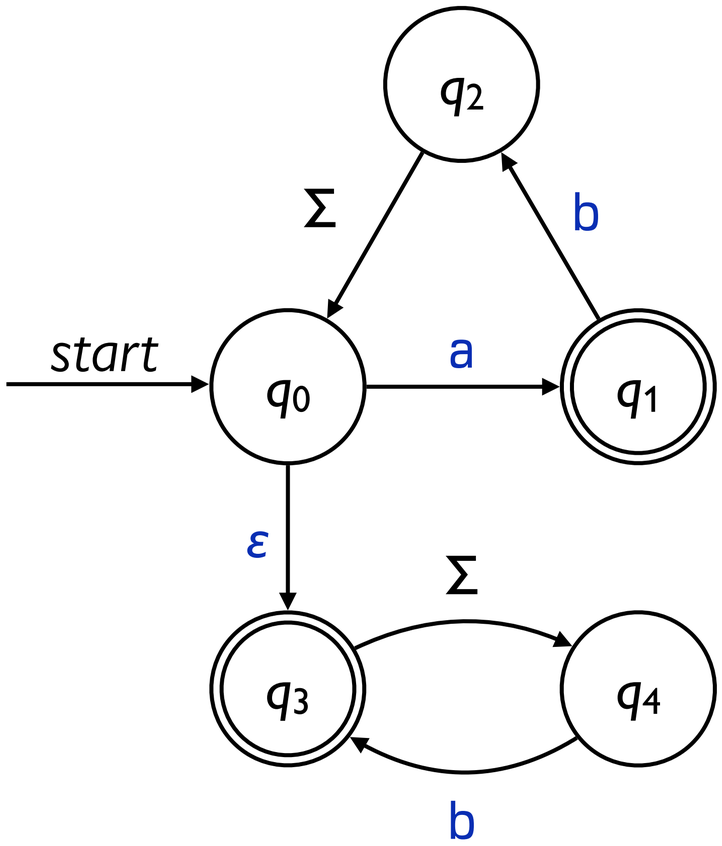

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | | |



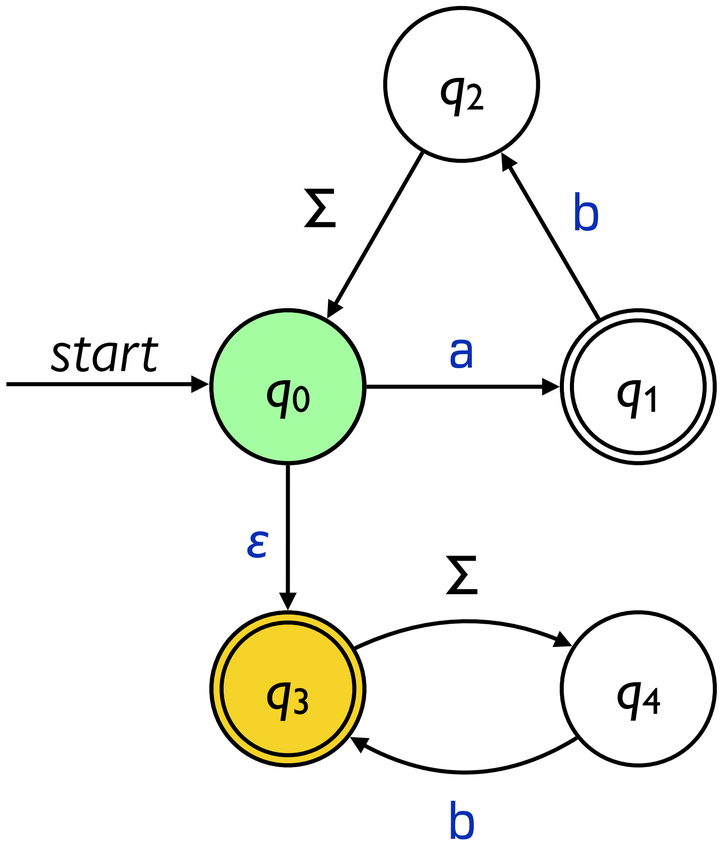

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | | |



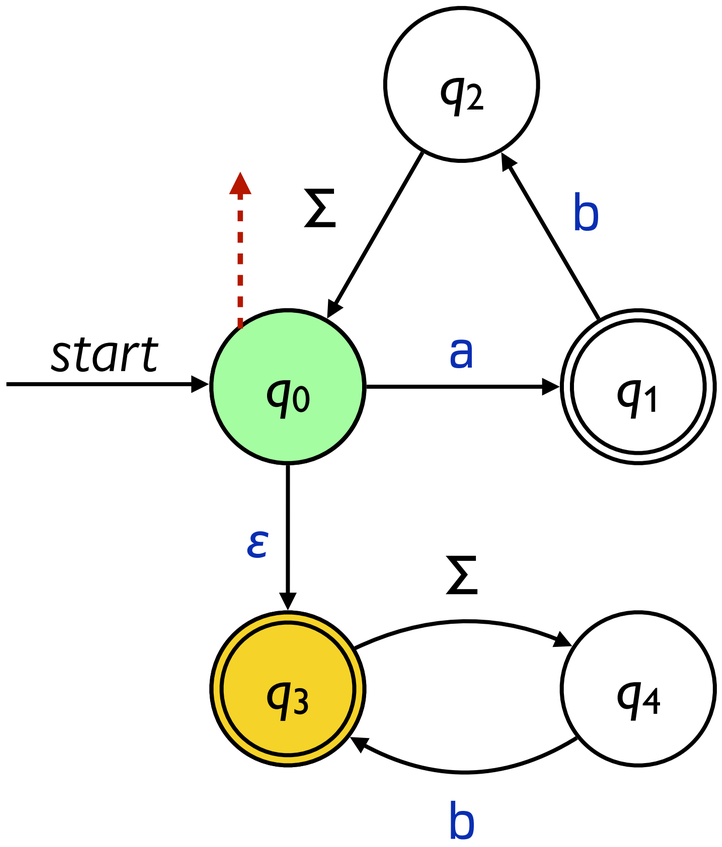

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



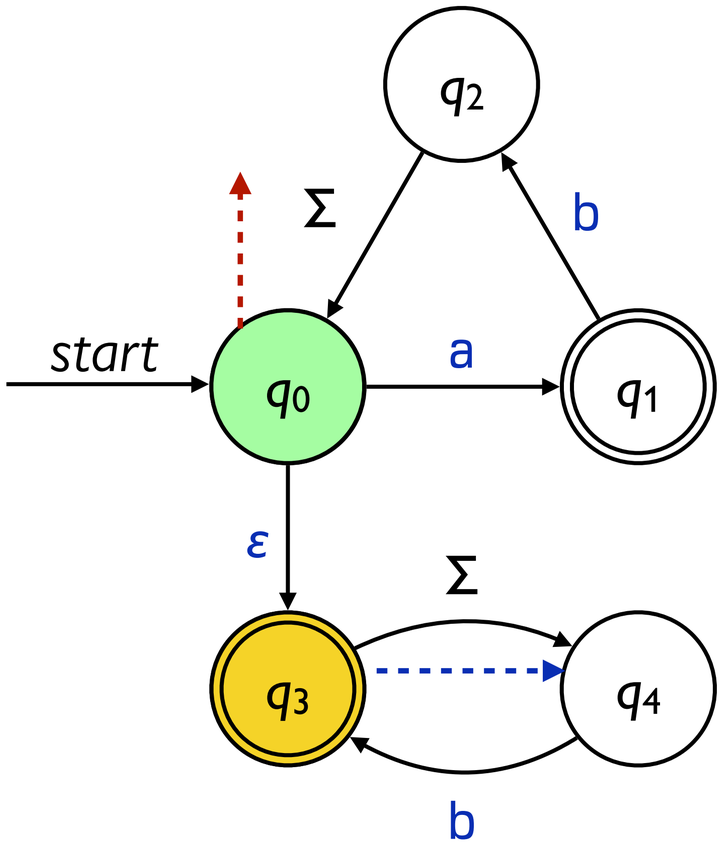

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



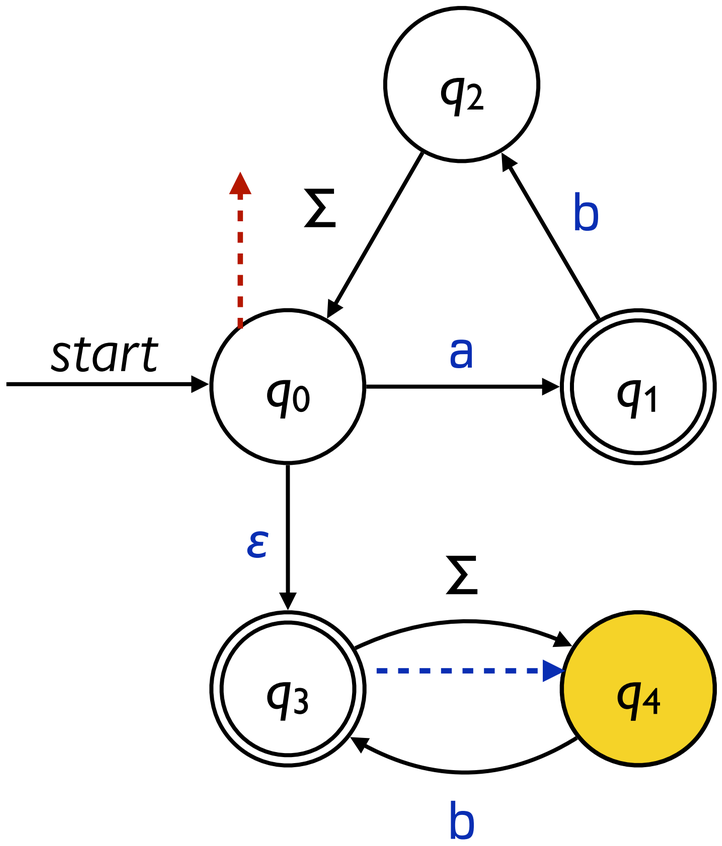

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



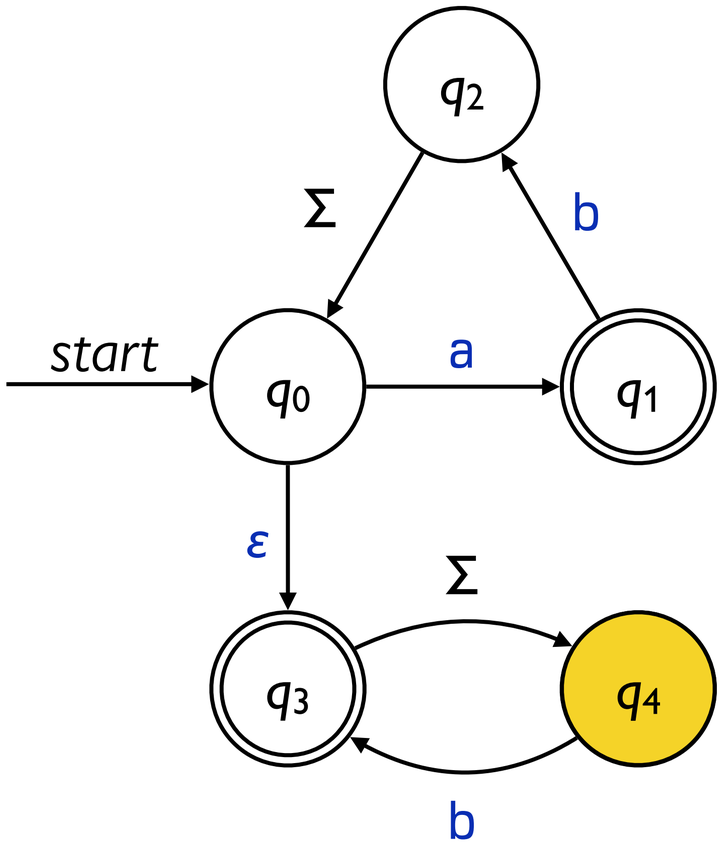

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



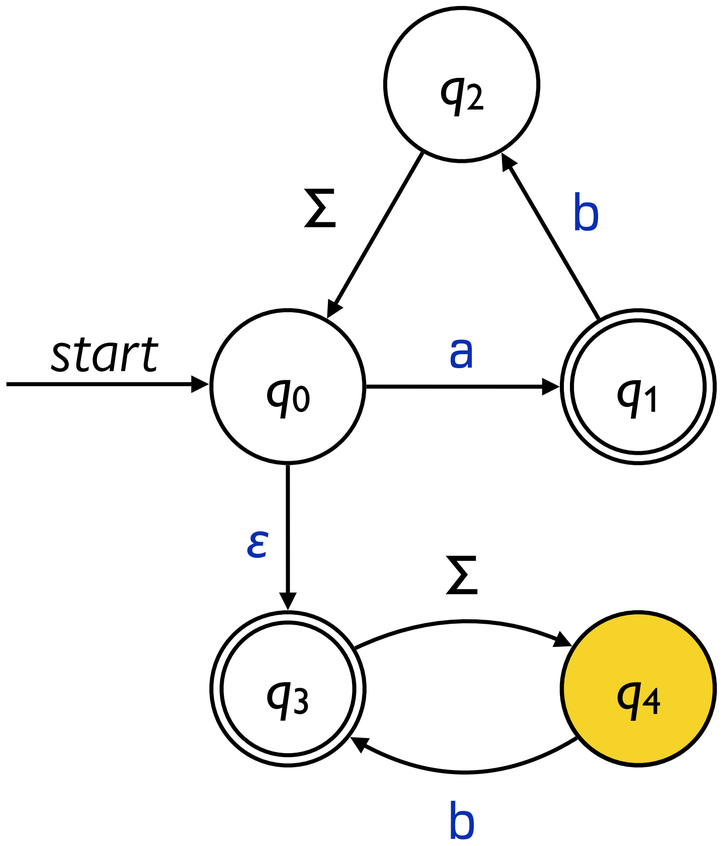

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



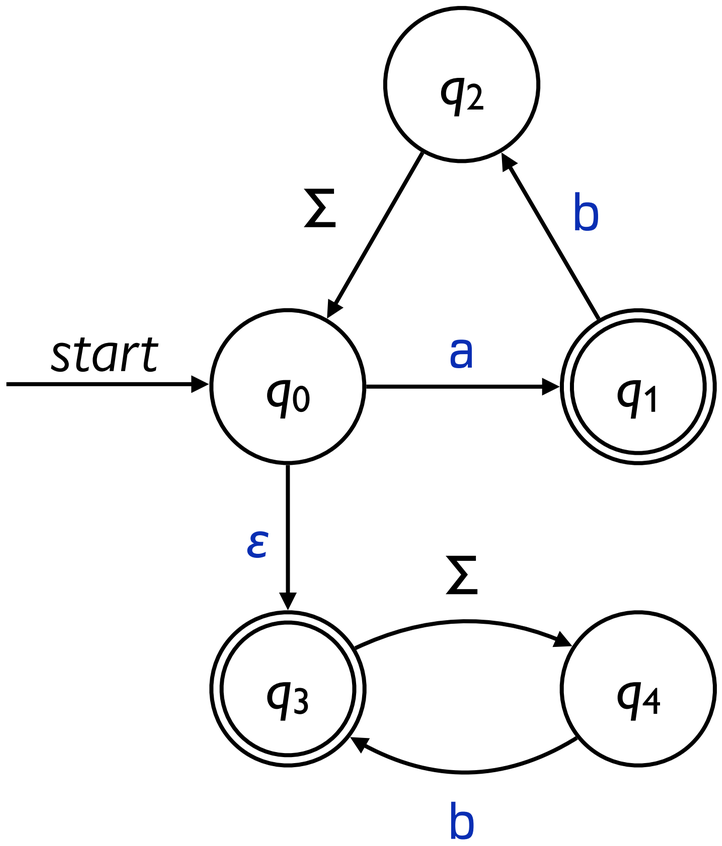

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



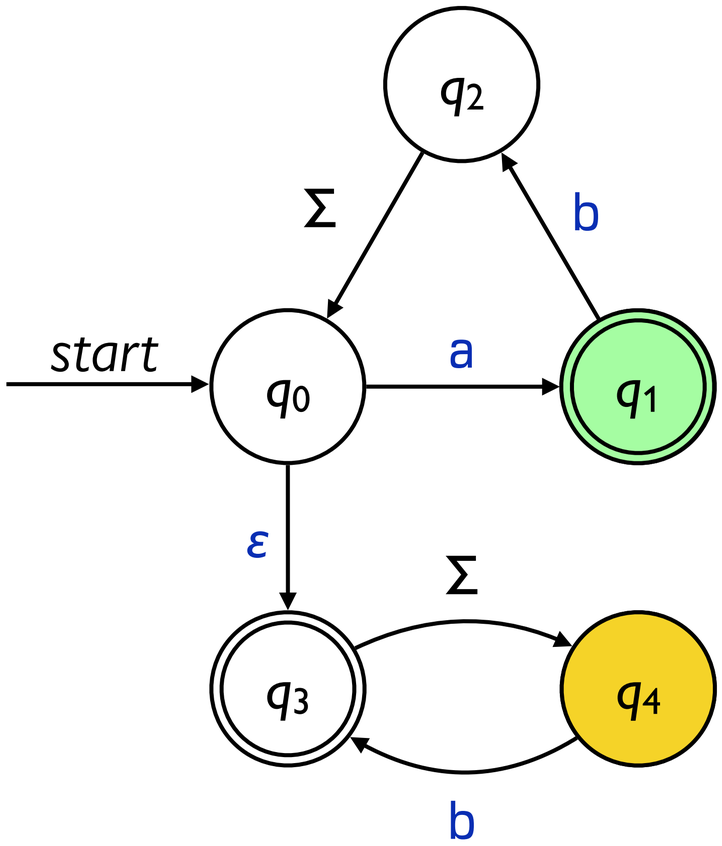

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | |



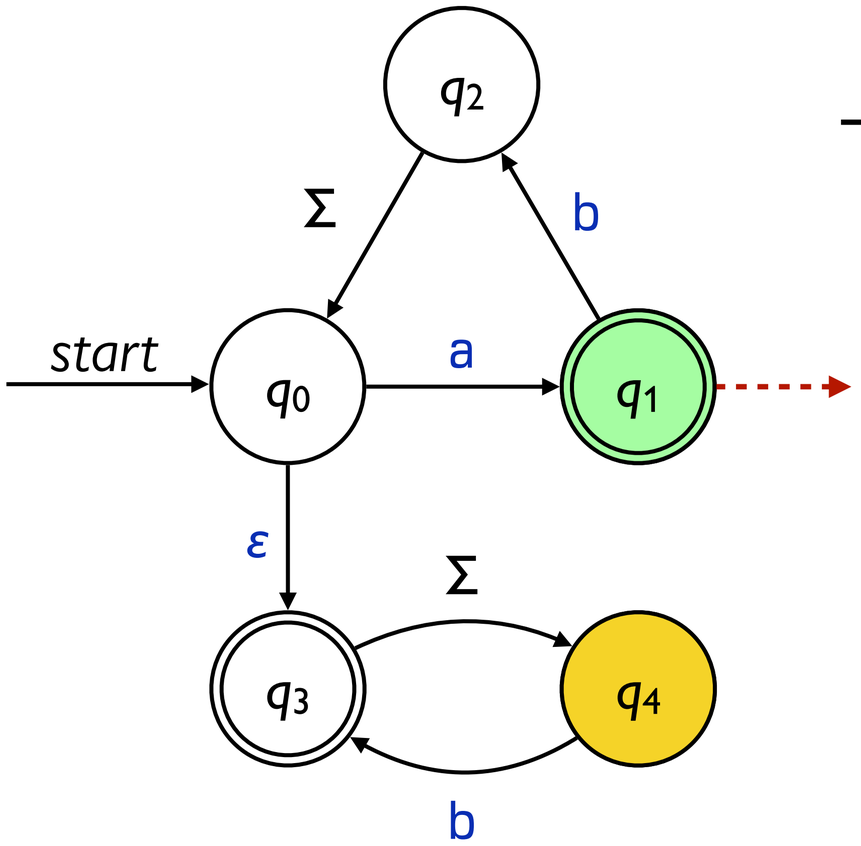

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |



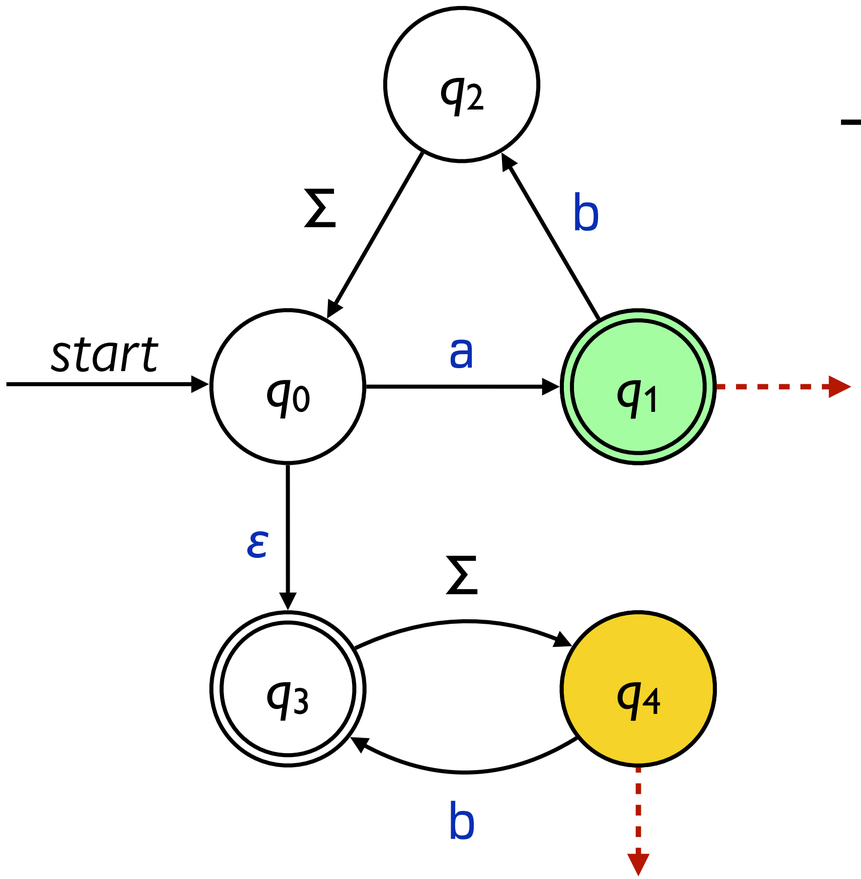

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |



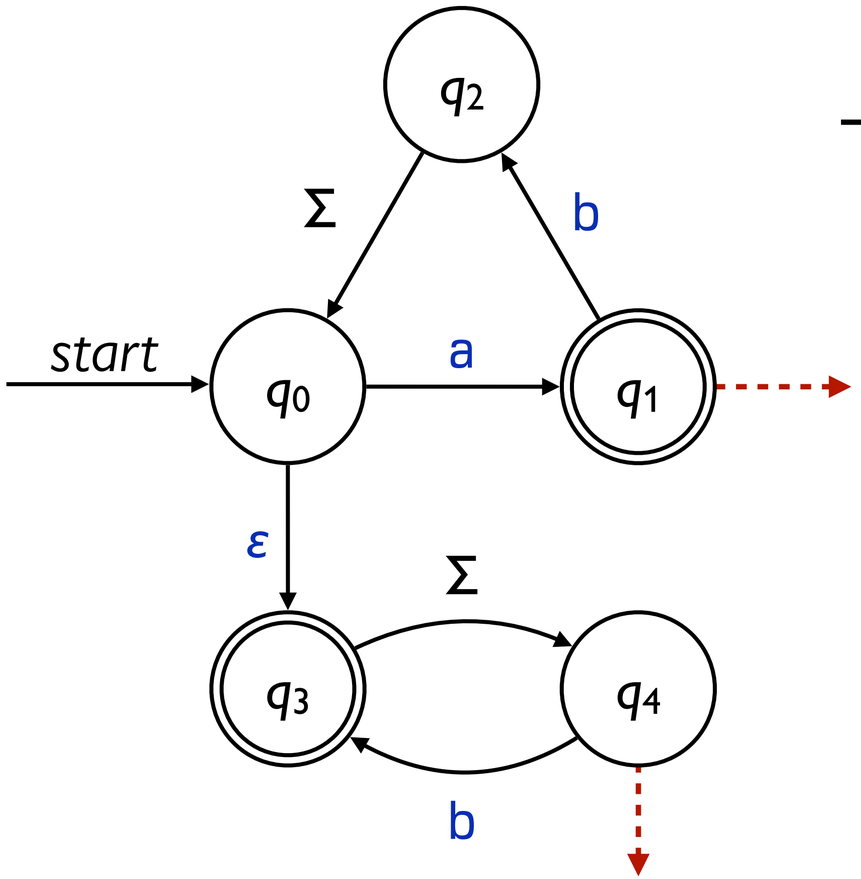

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | | |



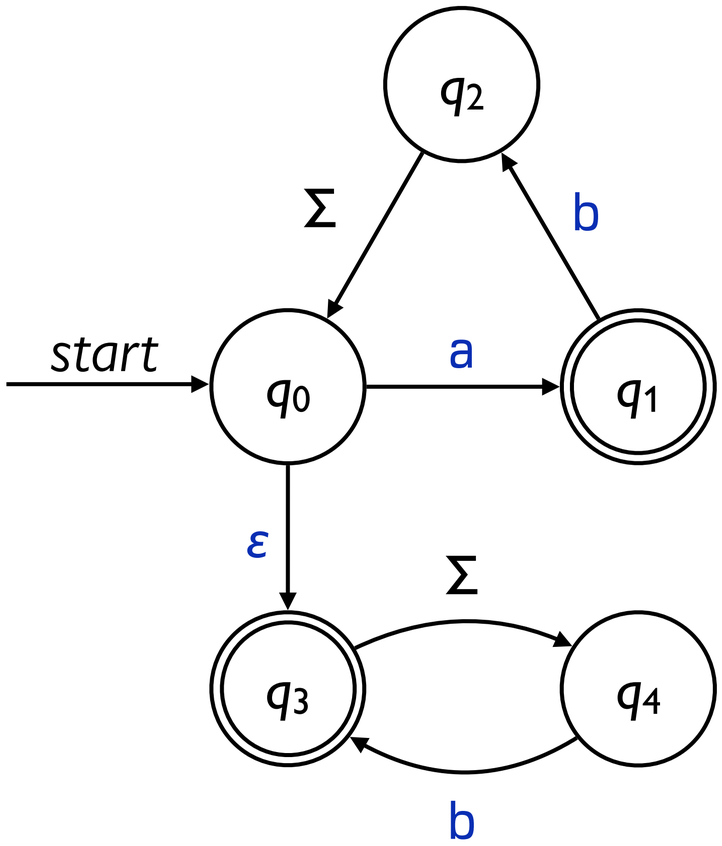

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| $\{q_1, q_4\}$ | | |



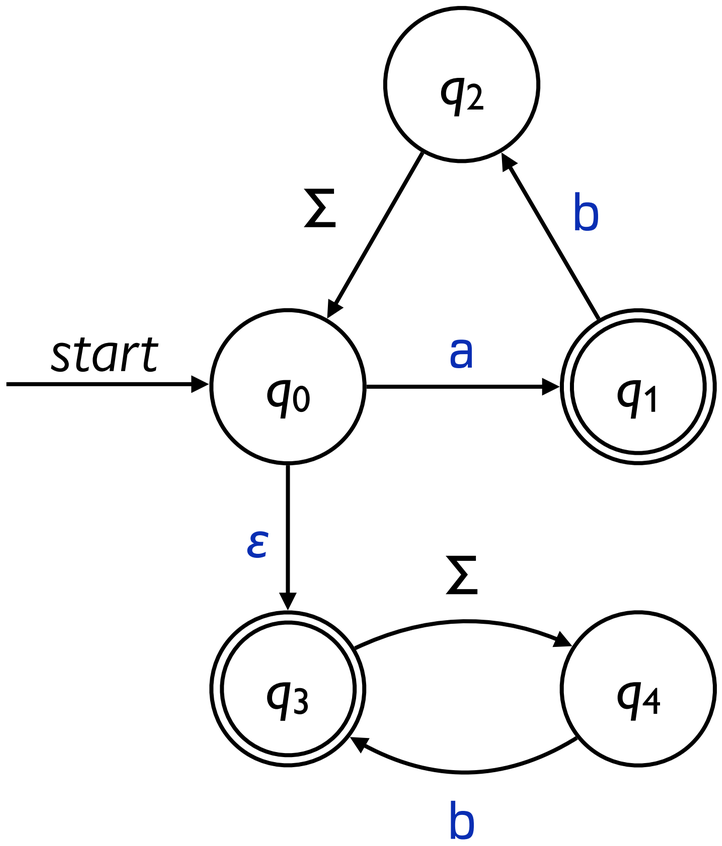

| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| $\{q_1, q_4\}$ | | |




| State | a | b |
|---|---|---|
| → $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| $\{q_1, q_4\}$ | | |

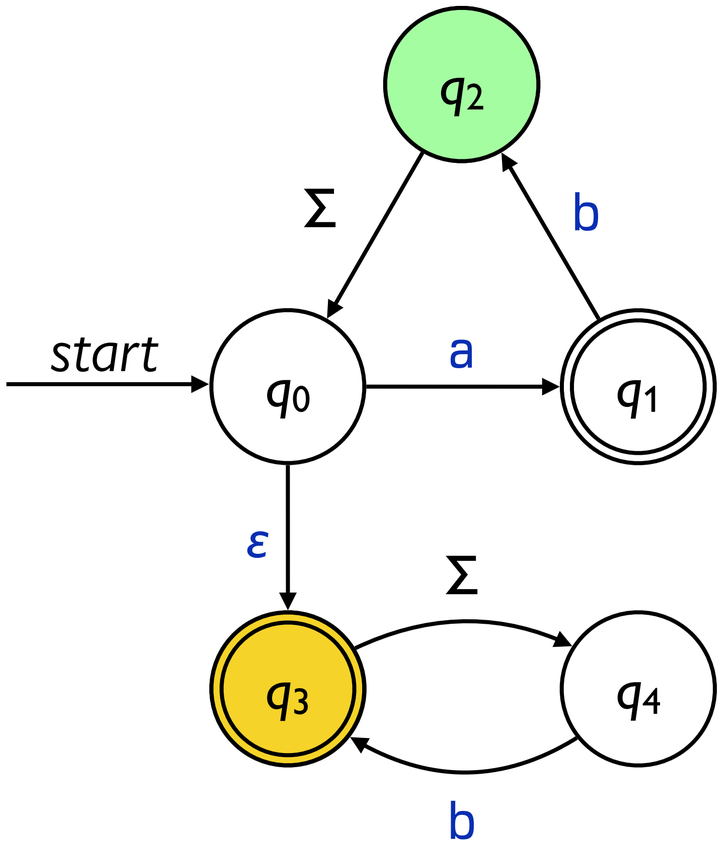



| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | |



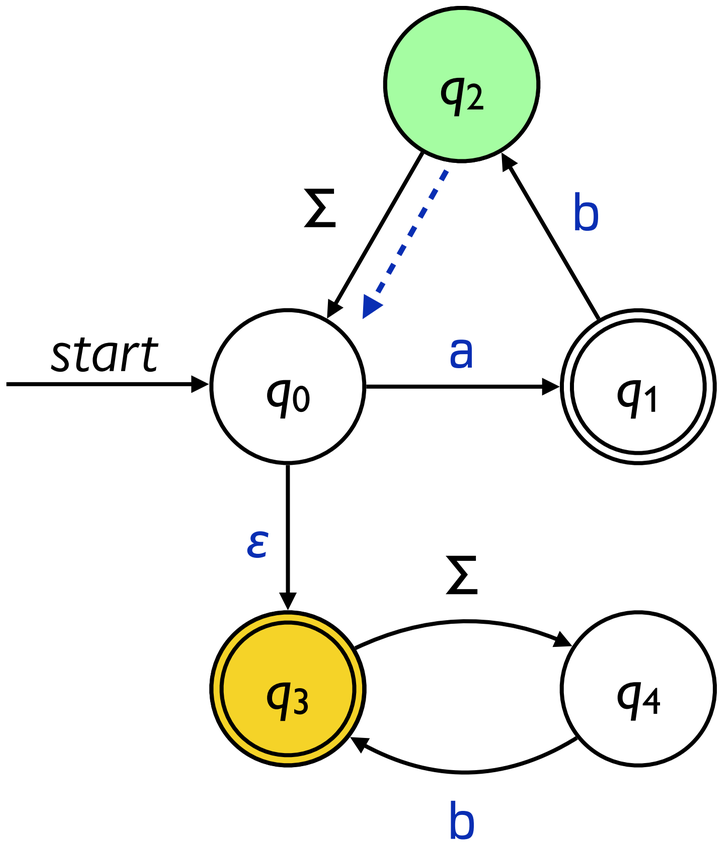

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | |

*A few steps later…*

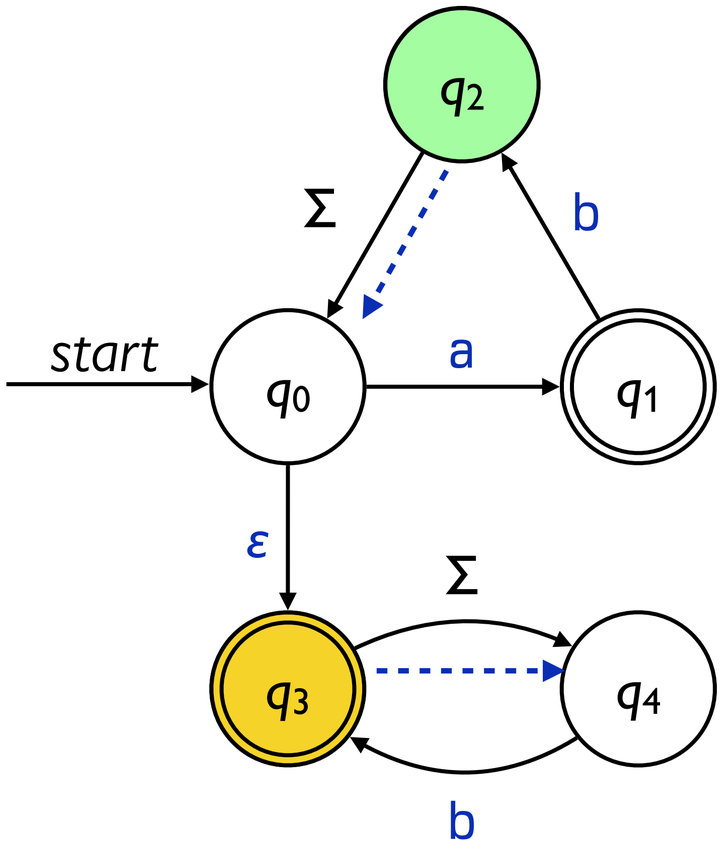



→

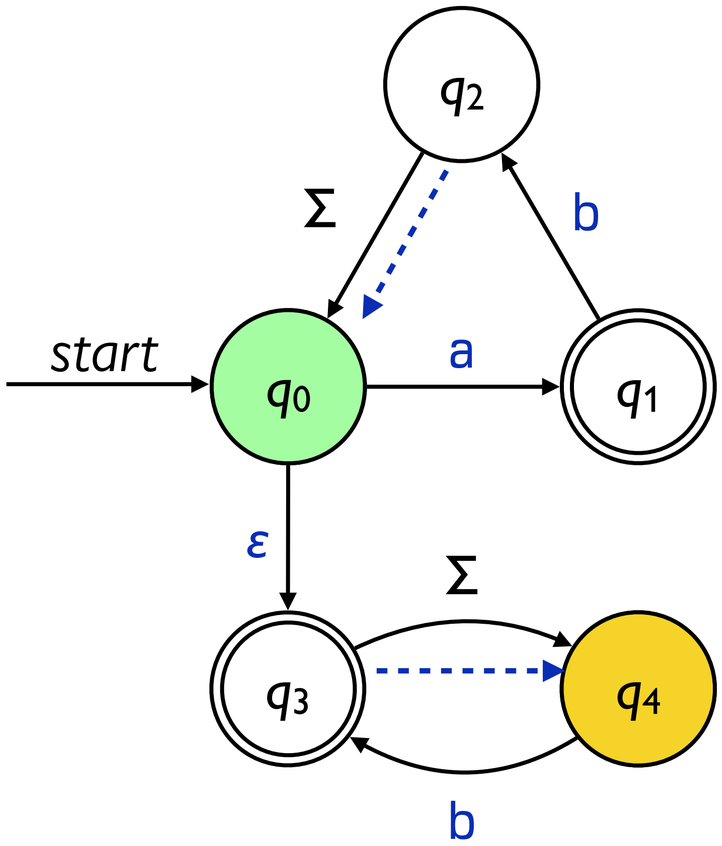
| State | a | b |
|---|---|---|
| $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| $\{q_2, q_3\}$ | | |



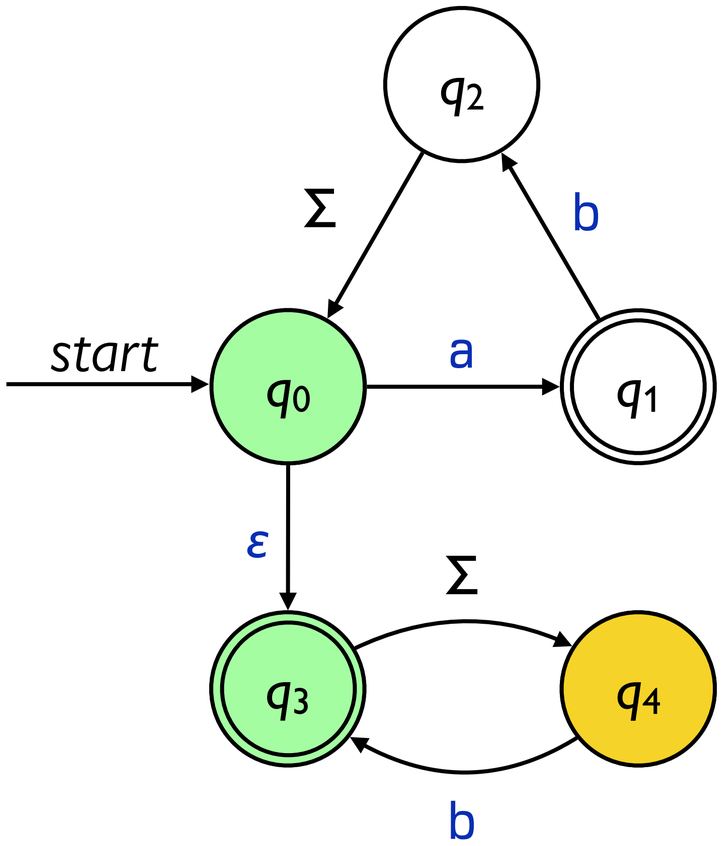

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | ∅ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | ∅ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | | |



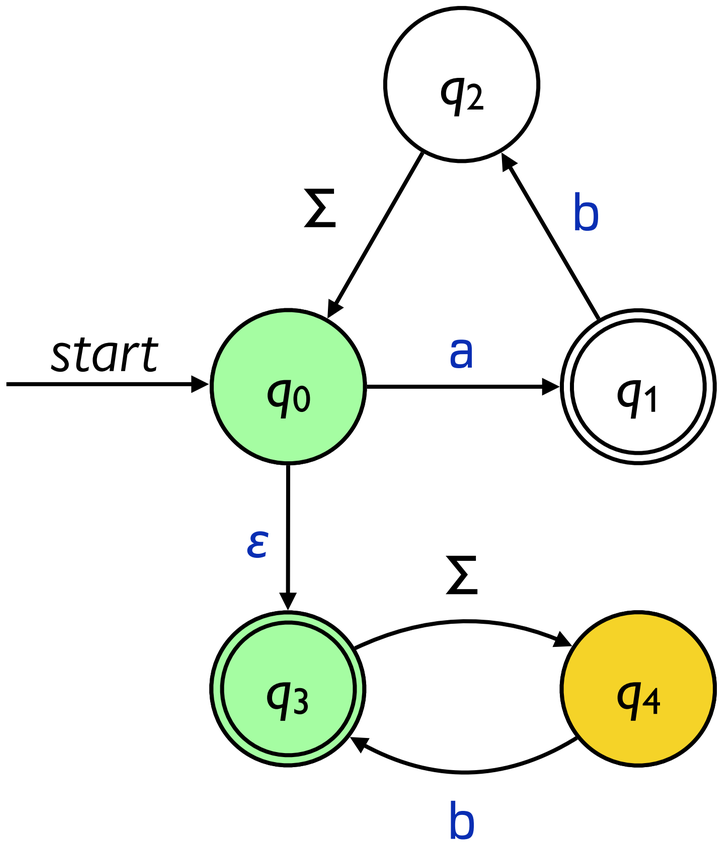

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | | |



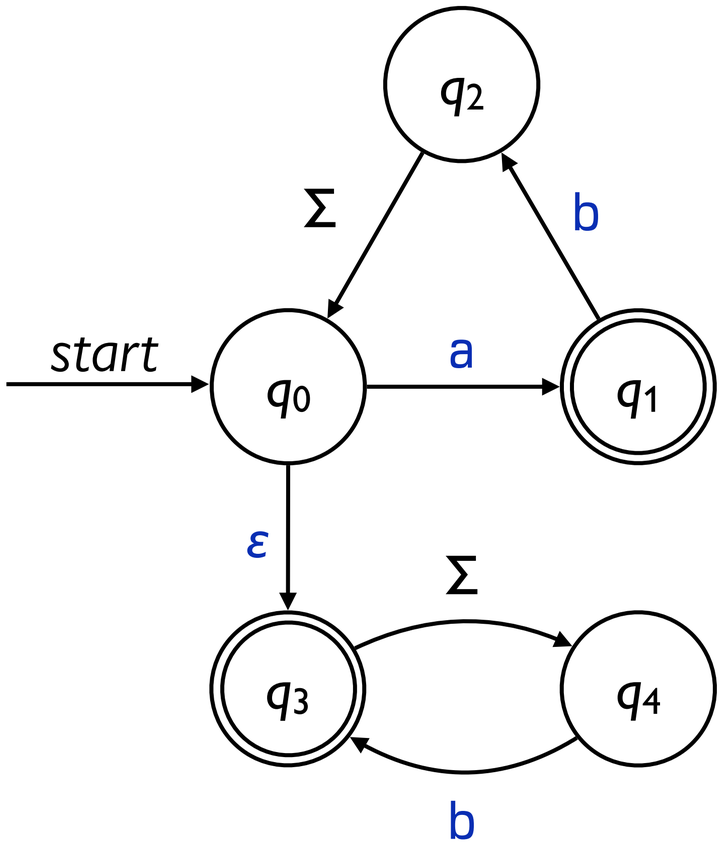

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | | |




| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | | |

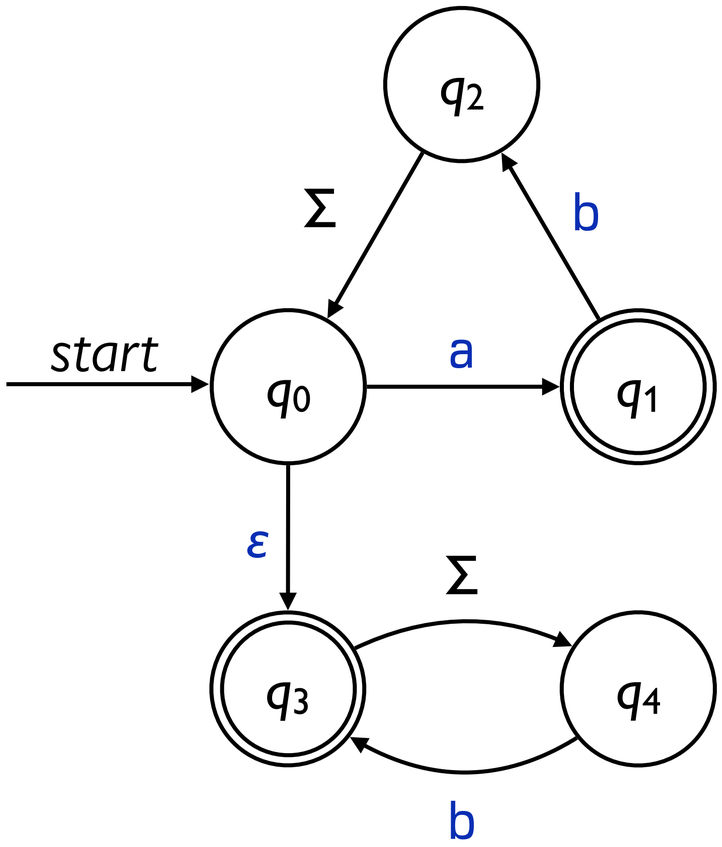



| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |



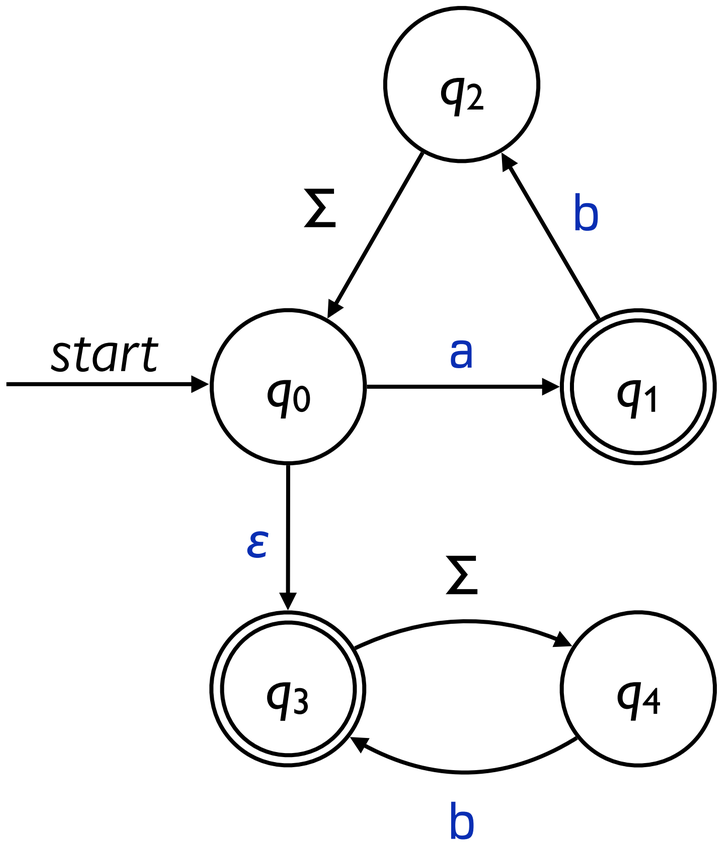

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |

*A few steps later…*

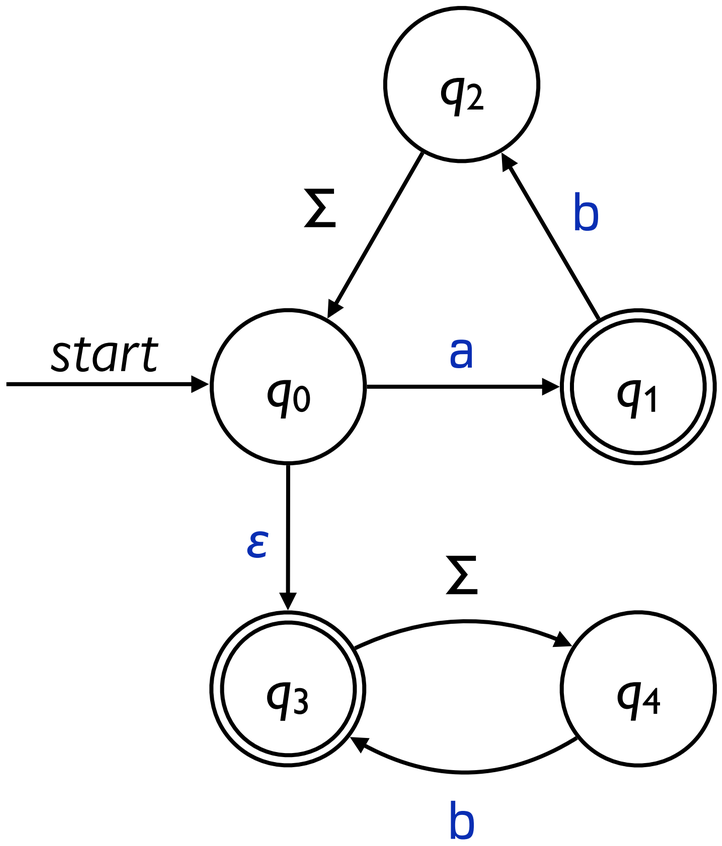



| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |
| | $\{q_3\}$ | $\{q_4\}$ | $\{q_4\}$ |
| | $\{q_0, q_3, q_4\}$ | $\{q_1, q_4\}$ | $\{q_3, q_4\}$ |
| | $\{q_3, q_4\}$ | $\{q_4\}$ | $\{q_3, q_4\}$ |

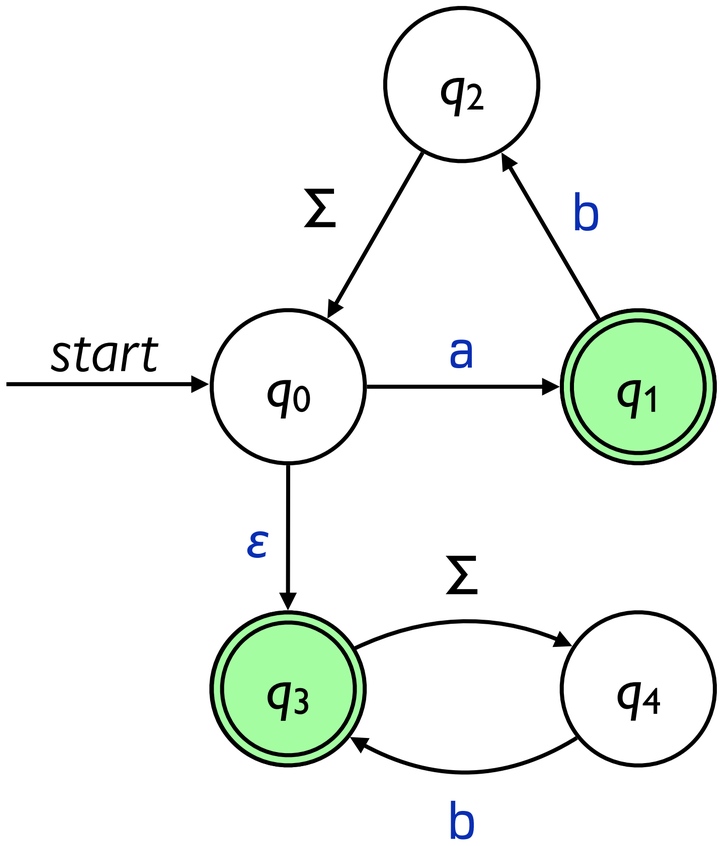
*What row is missing?*



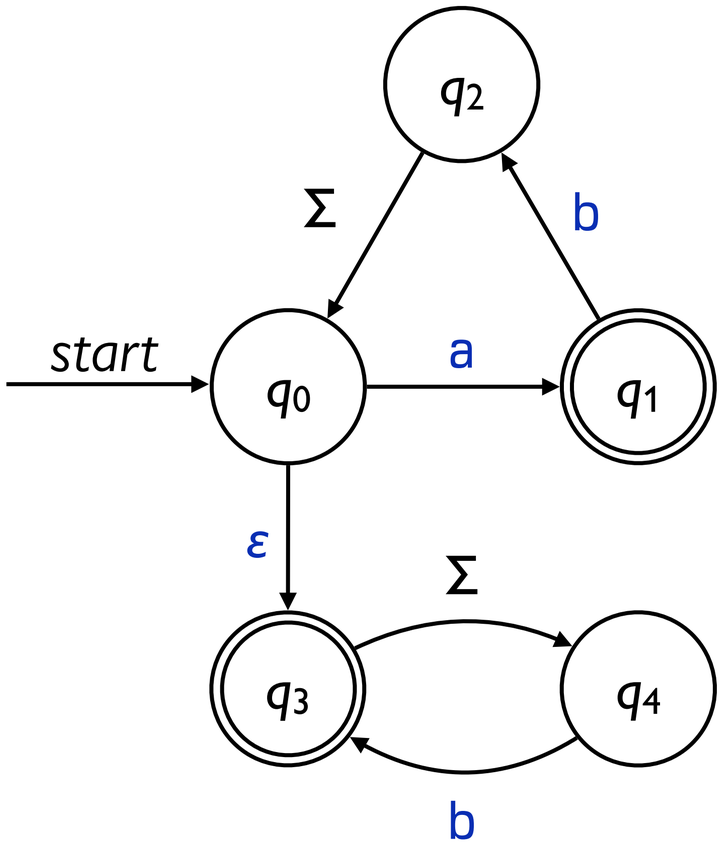

| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |
| | $\{q_3\}$ | $\{q_4\}$ | $\{q_4\}$ |
| | $\{q_0, q_3, q_4\}$ | $\{q_1, q_4\}$ | $\{q_3, q_4\}$ |
| | $\{q_3, q_4\}$ | $\{q_4\}$ | $\{q_3, q_4\}$ |
| | $\varnothing$ | | |




| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |
| | $\{q_3\}$ | $\{q_4\}$ | $\{q_4\}$ |
| | $\{q_0, q_3, q_4\}$ | $\{q_1, q_4\}$ | $\{q_3, q_4\}$ |
| | $\{q_3, q_4\}$ | $\{q_4\}$ | $\{q_3, q_4\}$ |
| | $\varnothing$ | $\varnothing$ | $\varnothing$ |




| | State | a | b |
|---|---|---|---|
| → | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |
| | $\{q_3\}$ | $\{q_4\}$ | $\{q_4\}$ |
| | $\{q_0, q_3, q_4\}$ | $\{q_1, q_4\}$ | $\{q_3, q_4\}$ |
| | $\{q_3, q_4\}$ | $\{q_4\}$ | $\{q_3, q_4\}$ |
| | $\varnothing$ | $\varnothing$ | $\varnothing$ |




| | State | a | b |
|---|---|---|---|
| →* | $\{q_0, q_3\}$ | $\{q_1, q_4\}$ | $\{q_4\}$ |
| * | $\{q_1, q_4\}$ | $\varnothing$ | $\{q_2, q_3\}$ |
| | $\{q_4\}$ | $\varnothing$ | $\{q_3\}$ |
| * | $\{q_2, q_3\}$ | $\{q_0, q_3, q_4\}$ | $\{q_0, q_3, q_4\}$ |
| * | $\{q_3\}$ | $\{q_4\}$ | $\{q_4\}$ |
| * | $\{q_0, q_3, q_4\}$ | $\{q_1, q_4\}$ | $\{q_3, q_4\}$ |
| * | $\{q_3, q_4\}$ | $\{q_4\}$ | $\{q_3, q_4\}$ |
| | $\varnothing$ | $\varnothing$ | $\varnothing$ |

# Creating a DFA from an NFA with ε-transitions

1. Compute the ***ε-closure*** for each state, i.e., the set of states reachable from that state following only ε-transitions.

2. The start state is the ε-closure of $q_0$, i.e., $E(\{q_0\})$

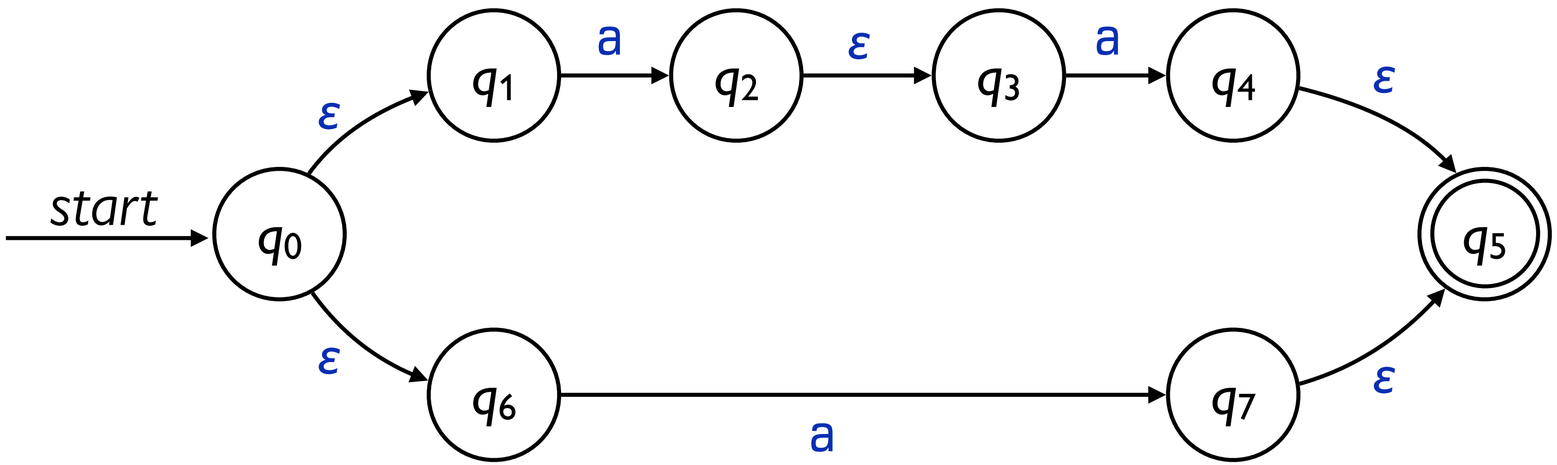
3. Define $\delta$ for each $a \in \Sigma$ and each ε-closed set $S$:

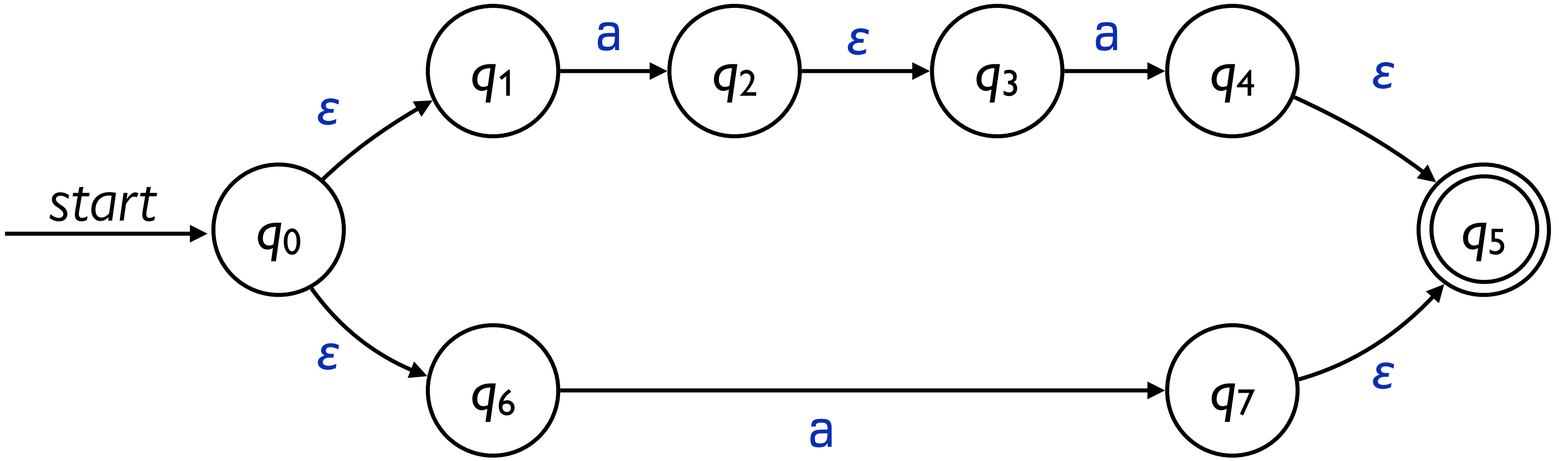
   If a state $p \in S$ can reach state $q$ on input $a$ (not ε!), then add a transition on input $a$ from $S$ to $E(q)$

4. The set of accept states for the DFA now includes those sets that contain at least one accept state of the NFA.

# Exercise

Convert this NFA to a DFA.






*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

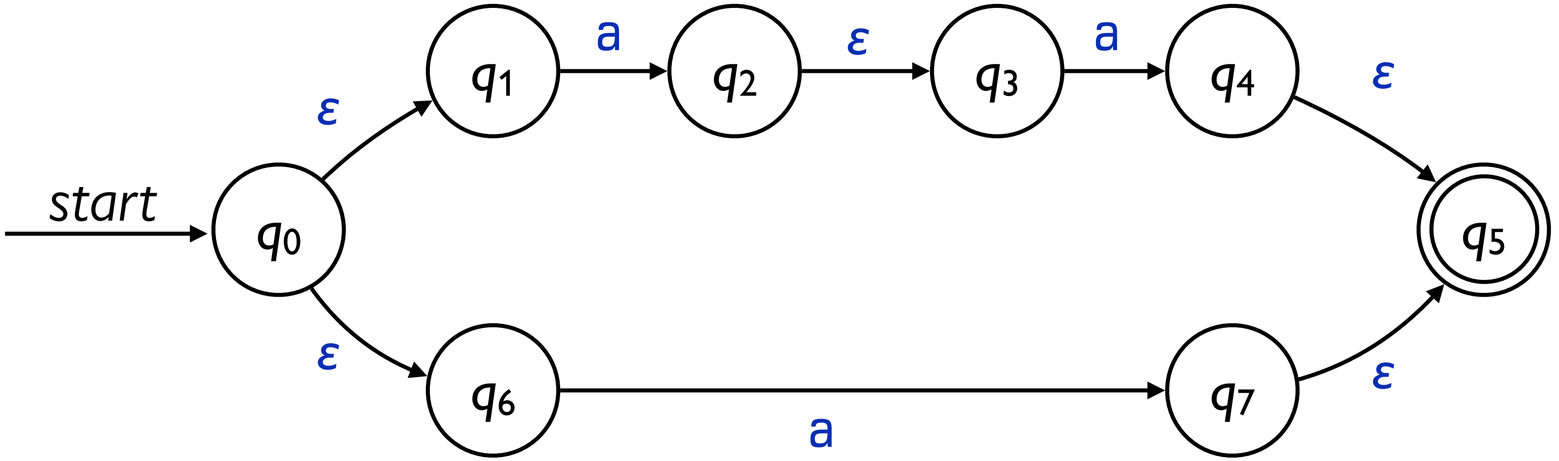
$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

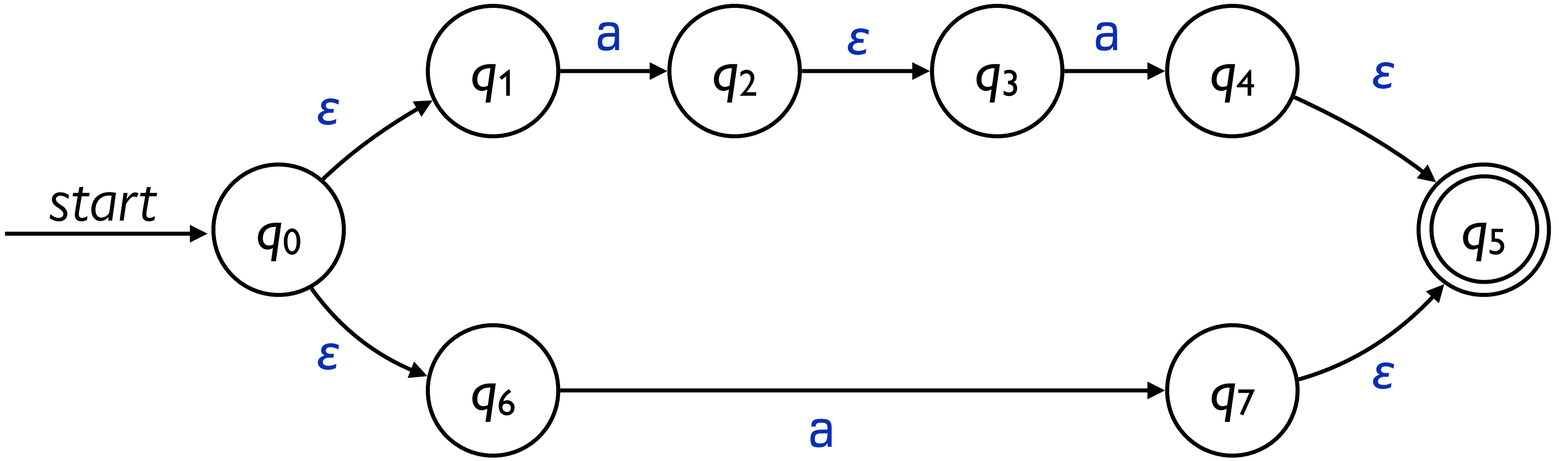
$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

*Start state*

| State |
| --- |
| → $\{q_0, q_1, q_6\}$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

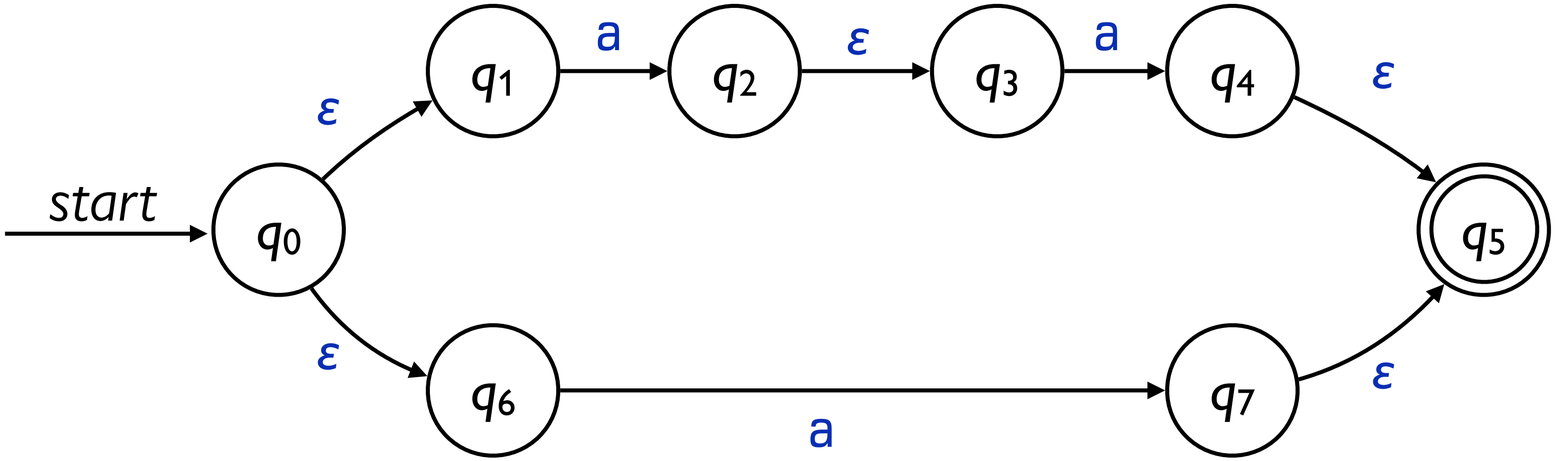
*Step 2*

*Start state*

*Step 3*

*Compute transitions, using ε-closure E.*

| State | a |
|---|---|
| → $\{q_0, q_1, q_6\}$ | $E(\{q_2, q_7\})$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

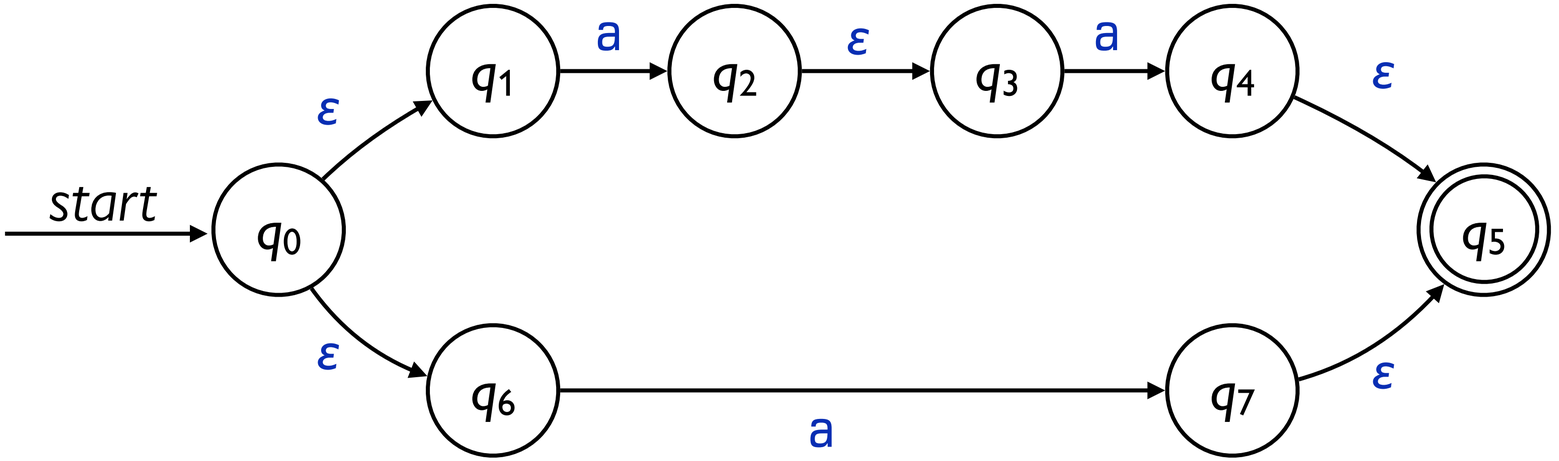
*Step 2*

*Start state*

*Step 3*

*Compute transitions, using ε-closure E.*

| State | a |
|---|---|
| → $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

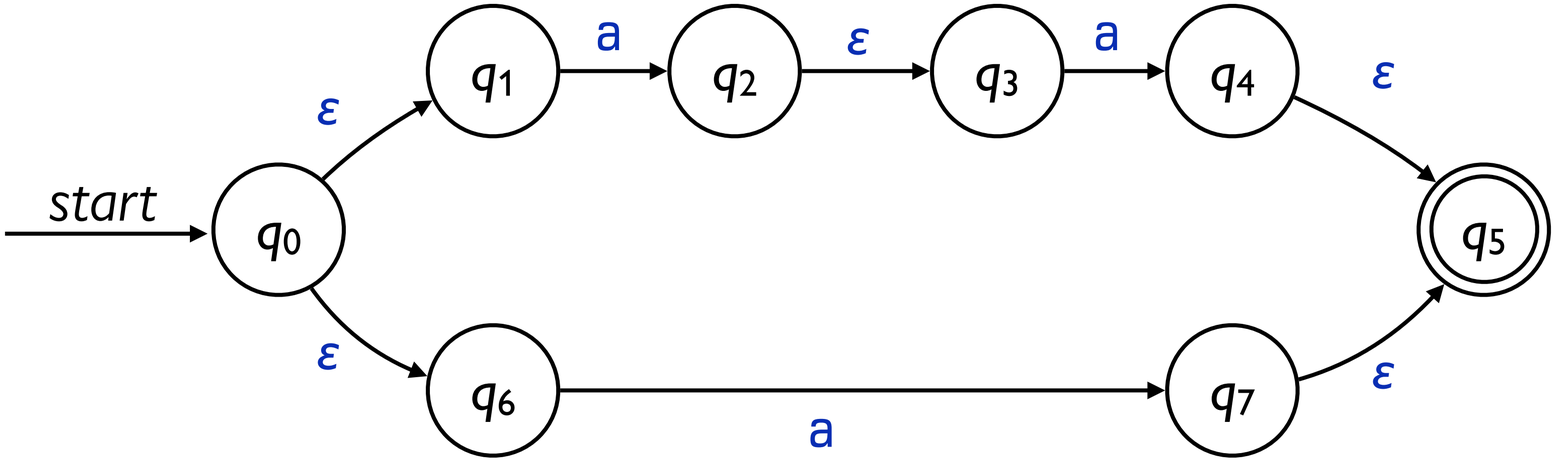
*Step 2*

*Start state*

*Step 3*

*Compute transitions, using ε-closure E.*

| State | a |
|---|---|
| → $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

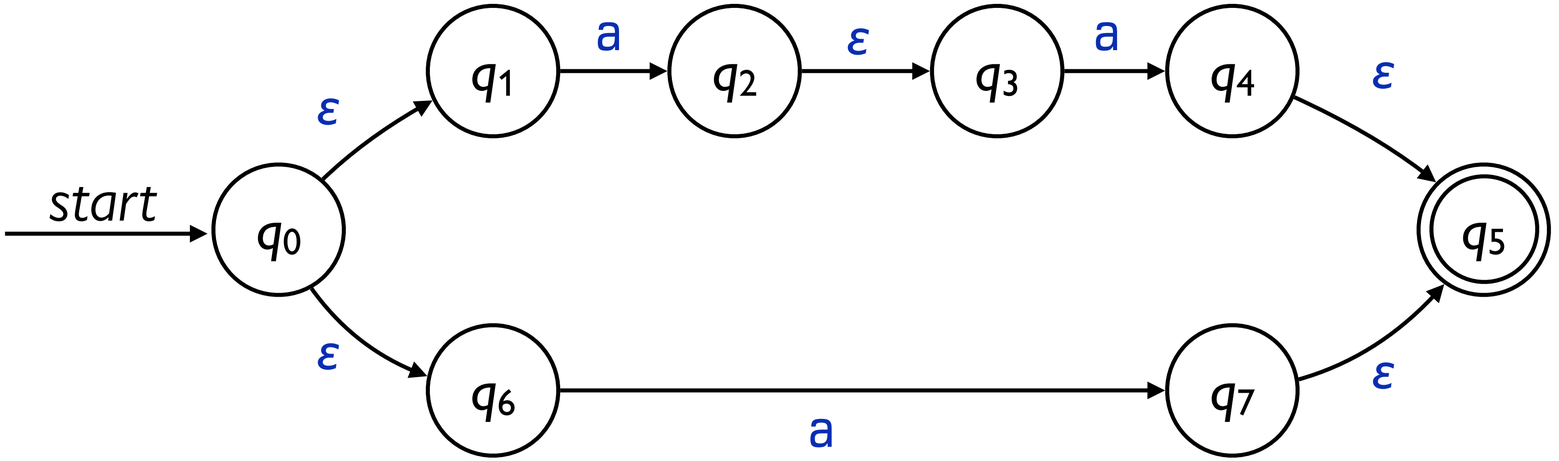
*Start state*

*Step 3*

*Compute transitions, using ε-closure E.*

→

| State | a |
|---|---|
| $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | $E(\{q_4\})$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

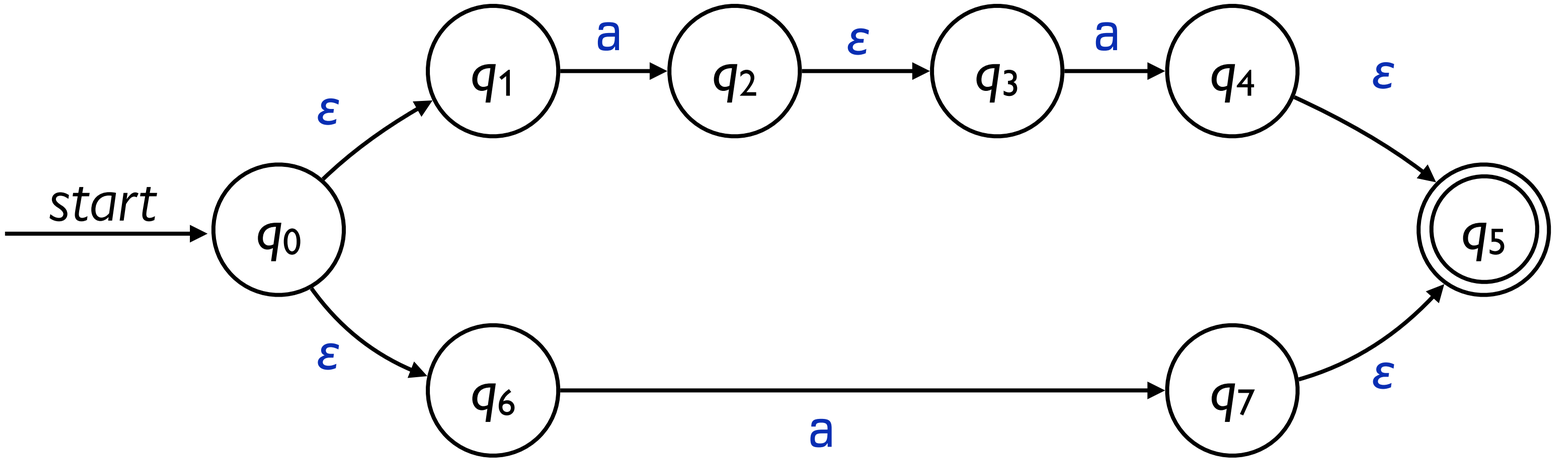
*Start state*

→

*Step 3*

*Compute transitions, using ε-closure E.*

| State | a |
|---|---|
| $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | $\{q_4, q_5\}$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

*Start state*

→ $\{q_0, q_1, q_6\}$

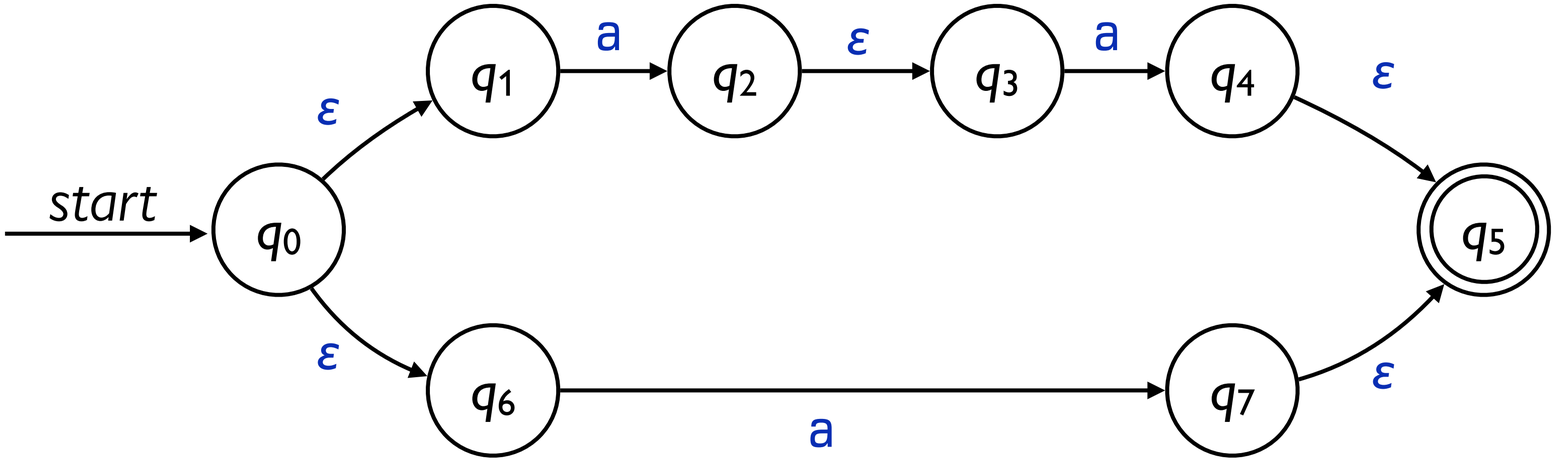
$\{q_2, q_3, q_7, q_5\}$

$\{q_4, q_5\}$

*Step 3*

*Compute transitions, using ε-closure E.*

| State | a |
|---|---|
| $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | $\{q_4, q_5\}$ |
| $\{q_4, q_5\}$ | |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

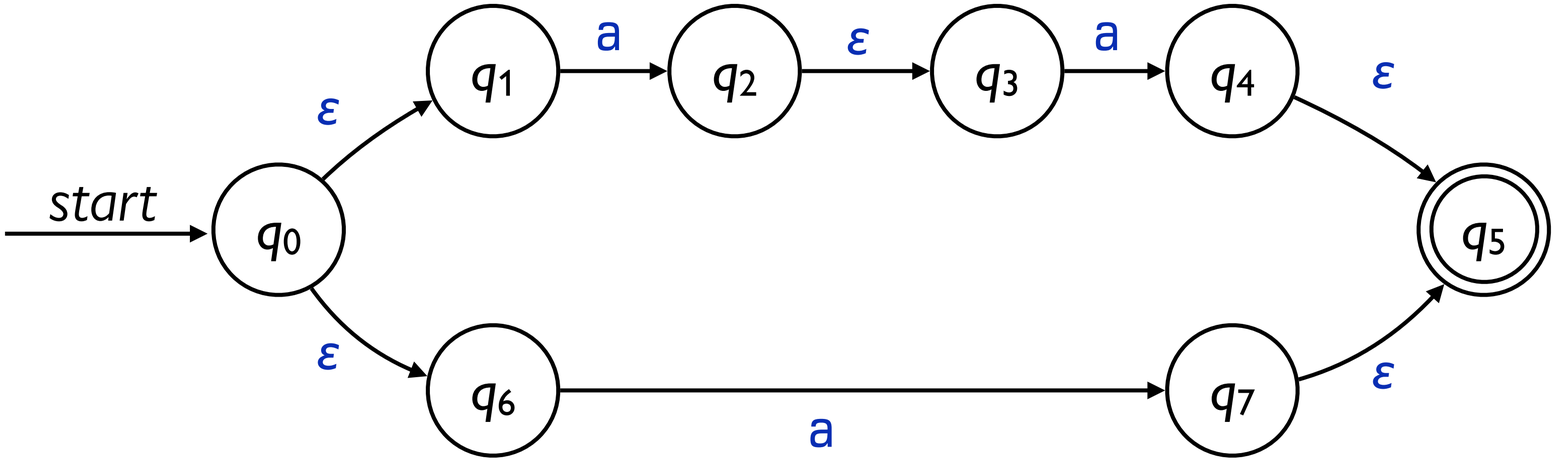
*Step 2*

*Start state*

*Step 3*

*Compute transitions, using ε-closure E.*

| State | a |
|---|---|
| → $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | $\{q_4, q_5\}$ |
| $\{q_4, q_5\}$ | $\varnothing$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

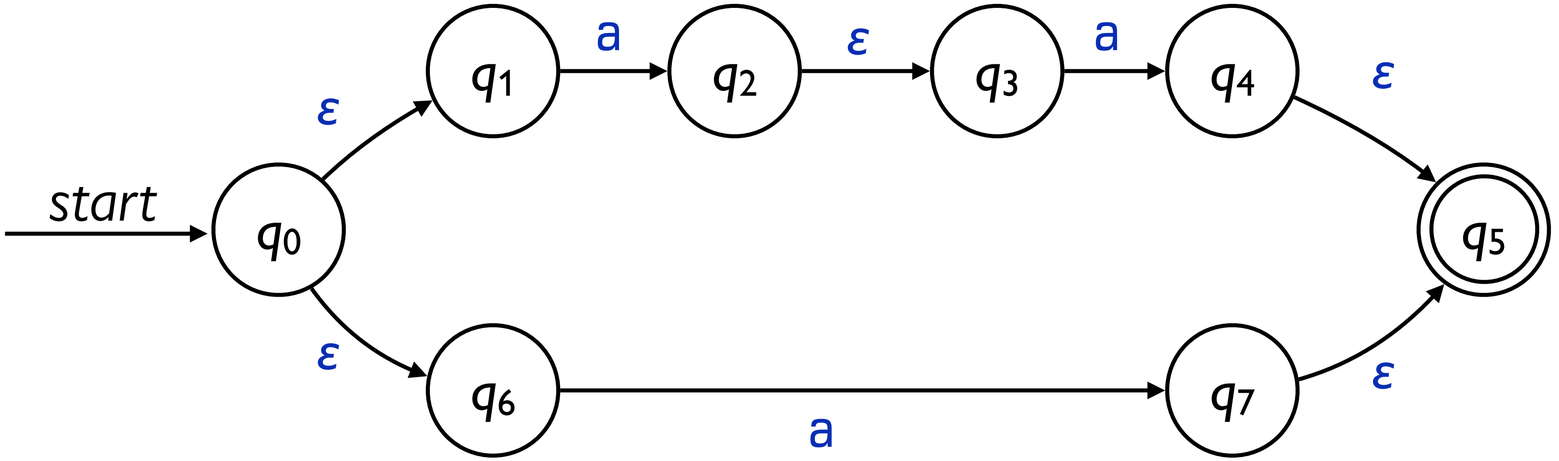
*Start state*

*Step 3*

*Compute transitions, using ε-closure E.*

→

| State | a |
|---|---|
| $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | $\{q_4, q_5\}$ |
| $\{q_4, q_5\}$ | $\varnothing$ |
| $\varnothing$ | $\varnothing$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

*Start state*

*Step 3*

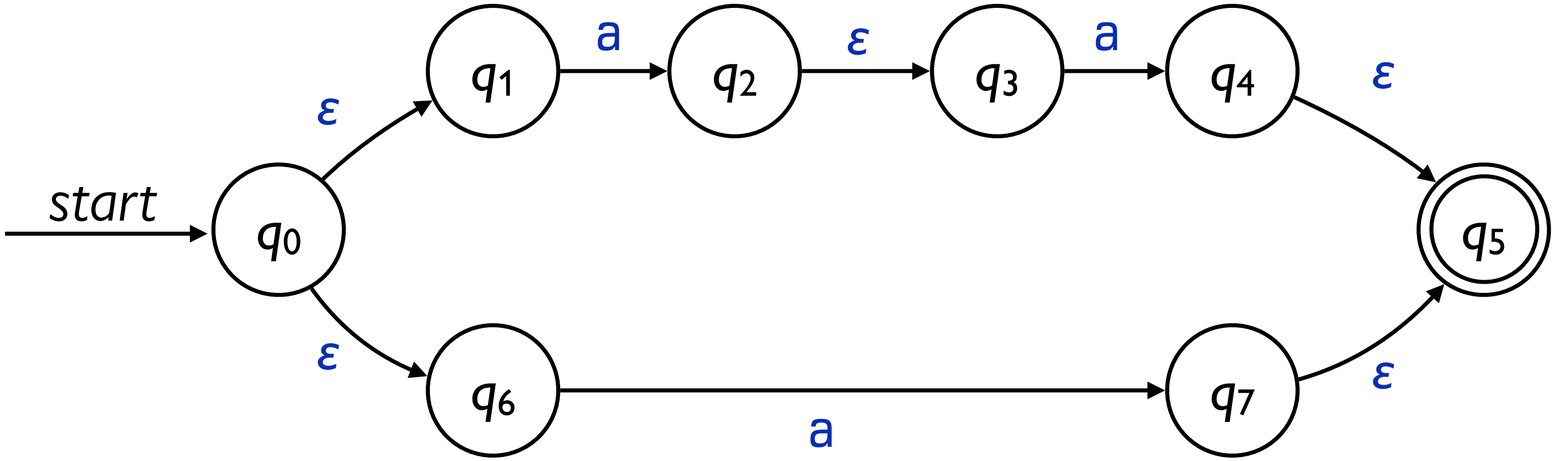
*Compute transitions, using ε-closure E.*

*Step 4*

*Accept states*

→

| State | a |
|---|---|
| $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| $\{q_2, q_3, q_7, q_5\}$ | $\{q_4, q_5\}$ |
| $\{q_4, q_5\}$ | $\varnothing$ |
| $\varnothing$ | $\varnothing$ |




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

*Start state*

*Step 3*

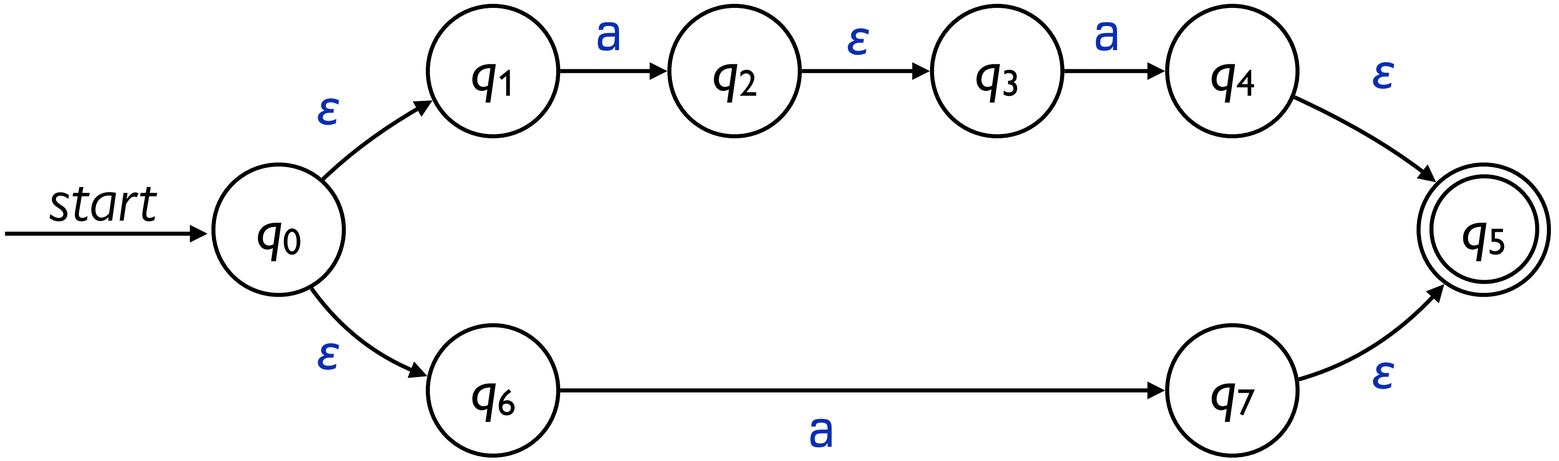
*Compute transitions, using ε-closure E.*

*Step 4*

*Accept states*

| | State | a |
|---|---|---|
| → | $\{q_0, q_1, q_6\}$ | $\{q_2, q_3, q_7, q_5\}$ |
| * | $\{q_2, q_3, q_7, q_5\}$ | $\{q_4, q_5\}$ |
| * | $\{q_4, q_5\}$ | ∅ |
| | ∅ | ∅ |

*This is the same process shown without using a table.*




*Step 1*

$E(\{q_0\}) = \{q_0, q_1, q_6\}$

$E(\{q_1\}) = \{q_1\}$

$E(\{q_2\}) = \{q_2, q_3\}$

$E(\{q_3\}) = \{q_3\}$

$E(\{q_4\}) = \{q_4, q_5\}$

$E(\{q_5\}) = \{q_5\}$

$E(\{q_6\}) = \{q_6\}$

$E(\{q_7\}) = \{q_7, q_5\}$

*Step 2*

*Start state:* $E(\{q_0\}) = \{q_0, q_1, q_6\}$

*Step 3*

$\delta(\{q_0, q_1, q_6\}, a) = E(\{q_2, q_7\}) = \{q_2, q_3, q_7, q_5\}$

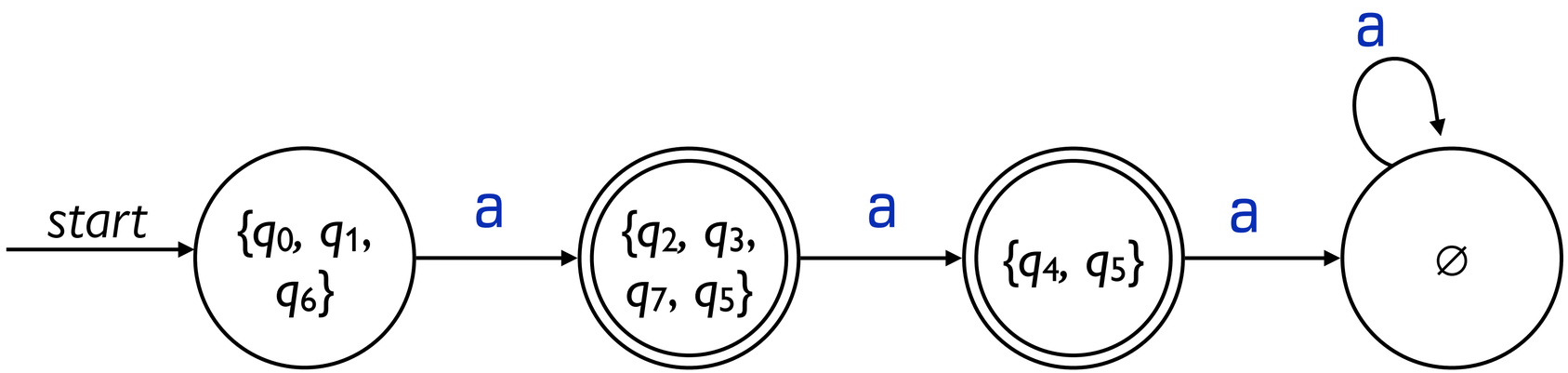
$\delta(\{q_2, q_3, q_7, q_5\}, a) = E(\{q_4\}) = \{q_4, q_5\}$

$\delta(\{q_4, q_5\}, a) = \varnothing$

$\delta(\varnothing) = \varnothing$

*Step 4*

$F = \{\{q_2, q_3, q_7, q_5\}, \{q_4, q_5\}\}$


start
{q0, q1, q6}
a
{q2, q3, q7, q5}
a
{q4, q5}
a
∅
a

# This method of transforming an NFA into a DFA is called the *subset construction*.

*a.k.a., power set construction*

Each state in the DFA corresponds to a set of states in the NFA.

The start state in the DFA corresponds to the start state of the NFA, *plus all states reachable via ε-transitions*.

The accept states in the DFA correspond to the sets of states that would be considered to accept in the NFA when using the massive parallel intuition.

If a state $q$ in the DFA corresponds to a set of states $S$ in the NFA, then the transition from state $q$ on a character $\alpha$ is found as follows:

Let $S'$ be the set of states in the NFA that can be reached by following a transition labeled $\alpha$ from any of the states in $S$.

*Let $S''$ be the set of states in the NFA reachable from some state in $S'$ by following zero or more ε-transitions.*

The state $q$ in the DFA transitions on $\alpha$ to a DFA state corresponding to the set of states $S''$.

*Now with ε-transitions!*

# Wrap-up

A language is called a *regular language* if there exists a DFA $D$ such that $L(D) = L$.

THEOREM A language $L$ is regular if and only if there is some NFA $N$ such that $L(N) = L$.

PROOF SKETCH If $L$ is regular, there exists some DFA for it, which we can easily convert into an NFA.

If $L$ is recognized by some NFA, we can use the subset construction to convert it into a DFA that recognizes the same language, so $L$ is regular.

We now have two perspectives on regular languages:

Regular languages are languages recognized by DFAs.

Regular languages are languages recognized by NFAs.

We can now reason about regular languages in two different ways, and we can use whichever model is more convenient.

This lecture incorporates material from:

Nancy Ide, Vassar College

Keith Schwarz, Stanford University

Michael Sipser, *Introduction to the Theory of Computation*

Jeffrey Ullman, Stanford University