Problem 1

Give short answers to each of the following. Be sure to adequately explain your answers for full credit.

a. Give an example of a regular language $R$ and a non-regular language $N$, such that $R \cap N$ is a regular language.

b. True or false: If an NFA with $n$ states accepts no string with length less than $n$, then it must accept no strings at all.

c. Consider this (false) claim: “If $L_1$ is a regular language and $L_1 \cup L_2$ is also regular, then $L_2$ must be regular”.

True or false: If the claim were true for all $L_1$ and $L_2$, then all languages would be regular.
Problem 2

Let $M$ be the NFA-ε below. The input alphabet is $\{a, b, c\}$.

Using the method described in the book and in class, construct a deterministic finite automaton that is equivalent to $M$. Show your steps.
Problem 3

Convert the following NFA-ε into an equivalent regular expression using the technique shown in class and on pages 72–76 of our book.

Step 1 (add new start and final states)

Step 2 (after q₀ removed)

Step 3 (after q₁ removed)

Step 4 (after q₂ removed) = Resulting regular expression
Problem 4

Using the construction method in the proof of Kleene's theorem in the textbook, construct a nondeterministic finite automaton that accepts the language \(((a \cup b)(a \cup b)^*a)^*\).
Problem 5

For each language, prove whether it is a regular language. If you say it is regular, give a DFA, NFA, or regular expression for \( L \). If you say it is not regular, prove this using the Pumping Lemma for regular languages and/or closure properties.

a. \( L = \{a^m b^m a^m b^n | m, n \geq 0\} \).

b. \( L = \{a^n | n \text{ is a multiple of 8}\} \)

c. \( L = \{x = y + z | x, y, z \text{ are binary integers and } x \text{ is the sum of } y \text{ and } z\} \). The alphabet is \( \Sigma = \{0, 1, =, +\} \); an example string is 1011 = 101 + 110.
Problem 6

Let $\Sigma = \{a, b\}$. Give a regular expression for the set of all strings in $\Sigma^*$ with exactly one occurrence of the substring $aaa$. 