1. Give tight asymptotic bounds for $T(n)$ in each of the following recurrences. Use backward substitution or either version of the Master Theorem. Assume that $T(n)$ is constant for $n \leq 2$. State your method of solution and if you use either version of the Master Theorem, state the case the recurrence falls into (Case 1, 2, or 3, etc.). Show all your work.

a. $T(n) = T\left(\frac{9n}{10}\right) + n$

b. $T(n) = 2T\left(\frac{n}{9}\right) + \sqrt{n}$

c. $T(n) = T(n-1) + n$

d. $T(n) = 3T\left(\frac{n}{2}\right) + n$

e. $T(n) = 6T\left(\frac{n}{4}\right) + n^2$

f. $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$
2. Consider the **Selection-Sort** algorithm, given below. Assume the input and output are as specified for the sorting problem on page 16 of our textbook.

**Selection-Sort**(A):
1. \( n = A.\text{length} \)
2. \( \text{for } j = 1 \text{ to } n - 1 \)
3. smallest = \( j \)
4. \( \text{for } i = j + 1 \text{ to } n \)
5. \( \text{if } A[i] < A[\text{smallest}] \)
6. smallest = \( i \)
7. exchange \( A[j] \) with \( A[\text{smallest}] \)

Consider the following loop invariant of the outer for loop (lines 2 through 7).

The algorithm maintains the loop invariant that at the start of each iteration of the outer for loop, the subarray \( A[1..j - 1] \) consists of the \( j - 1 \) smallest elements in the array \( A[1..n] \) and the subarray \( A[1..j - 1] \) is in sorted order.

Show that the loop invariant is true using a formal proof like that shown in class for **Insertion-Sort** (prove the invariant holds initially, at every subsequent iteration, and at termination, according to the value of \( j \)).