1. For the input keys $<10, 22, 31, 4, 15, 28, 17, 88, 59>$ hash function $h'(k) = k \mod 11$, and a hash table of length $m = 11$, illustrate the result of inserting these keys in the order shown using open addressing methods (a)--(c), below. For each case, state the number of collisions. (Show all your calculations and fill in the final table values in the hash tables below.)

(a) (5 points) Insert input keys $<10, 22, 31, 4, 15, 28, 17, 88, 59>$ using linear probing.

\[
\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\end{array}
\]

(b) (5 points) Insert input keys $<10, 22, 31, 4, 15, 28, 17, 88, 59>$, using quadratic probing with the $i^{th}$ probe $h(k, i) = (h'(k) + c_1 i + c_2 i^2) \mod m$, where $c_1 = 1$ and $c_2 = 3$.

\[
\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\end{array}
\]
(c) (5 points) Insert input keys \(<10, 22, 31, 4, 15, 28, 17, 88, 59>\), using double hashing with 
\(h_1(k) = h'(k), h_2(k) = 1 + (k \mod (m - 1)),\) and the \(i^{th}\) probe 
\(h(k, i) = (h_1(k) + h_2(k)i) \mod m.\)

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline \\
\end{array}
\]

2. (5 points) For the input keys \(<1, 12, 4, 43, 18, 22, 9, 81, 25, 7, 42, 16, 24>\) and hash function 
\(h'(k) = k \mod 11,\) show how the keys would be distributed if they are inserted in the order shown and 
when using chaining to resolve collisions. Fill in the table to depict the final array of lists. What is the 
load factor \(\alpha\) on the resulting table?

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline \\
\end{array}
\]