1. (5 points) Illustrate the operation of Counting-Sort on the array \( A = (6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2) \), using either the algorithm given in Chapter 8 of our textbook, or using the algorithm presented in the video for counting sort posted on our lectures link. Specify which algorithm you are using. Complete each step of the algorithm and redraw the array changes until the entire \( A \) array is sorted into the \( B \) array. No credit will be given if no work is shown.

2. (5 points) Using Figure 8.3 of our textbook and the video posted at the lectures link on our course webpage as a model, illustrate the operation of Radix-Sort on the following list of 4-digit numbers:

2142 0314 1220 1720 7142 1214 1142 9107 8519 6004 5558 3080 1940 3142 0542

Rewrite all the numbers as each column is sorted in your solution. You do not have to depict the action of counting sort on each column.
3. (2 points) What is the largest possible number of internal nodes in a red-black tree with black-height 4? Give an example of such a red-black tree, labeling nodes with actual keys and indicating whether the nodes are red or black. In general, what is the largest possible number of internal nodes in a red-black tree of black-height $k$? Note that the parent nodes of NIL leaves are considered to be internal nodes.

4. (10 points) Insert the keys 5, 6, 8, 3, 2, 4, 7 into an AVL tree. Show the tree after each insertion. Label each imbalance as LL, LR, RR, RL and redraw the tree after each rotation. For a double rotation that occurs after an LR imbalance, for example, show the tree after the left rotation and again after the right rotation, using arrows to indicate which nodes will move, as shown in class.
5. (10 points) Show the red-black trees that result from successively inserting the keys 8, 19, 12, 31, 3, 14 into an initially empty red-black tree. See page 315 of our textbook for RB-INSERT and page 316 for RB-INSERT-FIXUP. Also, watch the videos about red-black trees posted on the lectures link on our class website. Show the color and the black-height at each node and redraw the tree after every new node is inserted and after every rotation. Write down any Red-Black Tree violations that occur, and mention any recoloring and rotations as shown in class on 10/31/19. You can show black nodes with a single circle and red nodes with a double circle in lieu of using colors (but using colors is fine).
6. (10 points) Insert the keys 5, 6, 8, 3, 2, 9, 7 into a 2-3 tree. Show the tree after each insertion. Label each promotion and split and redraw the tree after each restructuring.