You should solve each of the problems in this assignment by programming and printing the programs and the solutions. You should use the bottom-up, dynamic programming algorithms in your solutions to these problems. Unless otherwise cleared with your professor, write your programs in Java. You will be graded on the correctness and output of your solution. Please do not copy your programs off the web.

1. (10 points) Using Floyd’s algorithm shown in the class notes for April 22nd, print the solution starting with the weight matrix shown below:

   \[
   D^{[0]} = \\
   \begin{pmatrix}
   0 & 2 & \infty & 1 & 8 \\
   6 & 0 & 3 & 2 & \infty \\
   \infty & \infty & 0 & 4 & \infty \\
   \infty & \infty & 2 & 0 & 3 \\
   3 & \infty & \infty & \infty & 0
   \end{pmatrix}
   \]

   Label each of the \(D\) and \(\Pi\) matrices as shown in the notes and in the textbook on page 696.

   (a) Print all matrices \(D^{[0]}\) through \(D^{[5]}\). \((D^{[0]}\) is given above.)

   (b) Print all matrices \(\Pi^{[0]}\) through \(\Pi^{[5]}\).

   After getting your program to work on the matrix above, test it on the example given on page 696 of our textbook to see if your answers are the same as the ones shown.

2. This question involves the matrix-chain multiplication problem. Suppose you want to find the optimal way to parenthesize 4 matrices of the following sizes:

   \[
   \begin{array}{c|cccc}
   \text{matrix} & A_1 & A_2 & A_3 & A_4 \\
   \text{dimension} & 40 \times 20 & 20 \times 30 & 30 \times 10 & 10 \times 30 \\
   \end{array}
   \]

   Input: \(p = \langle 40, 20, 30, 10, 30 \rangle\)

   Using the \textsc{Matrix-Chain-Order} algorithm from our lecture notes and from pages 375–377 of our textbook, compute a way to parenthesize the set of matrices such that the least number of multiplications are used. Your program should return the \(m\) and \(s\) tables.

   Provide the \(s\) table your program computes as input to the \textsc{Print-Optimal-Parens} algorithm, with this call: \textsc{Print-Optimal-Parens}(s, 1, 4), printing the optimal way to parenthesize matrices \(A_1\) through \(A_4\).

   After running your program on the matrix chain above, test it on the problem given on page 376 of our textbook to check if your answers are the same as the ones shown.