Red Black Deletion Steps

Initial Steps

Pick one

- If the node we deleted has 2 NIL children, its replacement $x$ is NIL.
- If the node we deleted has 1 NIL child and 1 non-NIL child, its replacement $x$ is the non-NIL child.
- If the node we deleted has 2 non-NIL children, set $x$ to the replacement's right child before the replacement is spliced out.

Pick one

- If the node we deleted is red and its replacement is red or NIL, we are done.
- If the node we deleted is red and its replacement is black, color the replacement red and proceed to the appropriate case.
- If the node we deleted is black and its replacement is red, color the replacement black. We are done.
- If the node we deleted is black and its replacement is NIL or black, proceed to the appropriate case.

Cases

0. Node $x$ is red

1. Node $x$ is black and its sibling $w$ is red

2. Node $x$ is black and its sibling $w$ is black and both of $w$'s children are black

3. Node $x$ is black and its sibling $w$ is black and
   - If $x$ is the left child, $w$'s left child is red and $w$'s right child is black
   - If $x$ is the right child, $w$'s right child is red and $w$'s left child is black

4. Node $x$ is black and its sibling $w$ is black and
   - If $x$ is the left child, $w$'s right child is red
   - If $x$ is the right child, $w$'s left child is red
Case 0: Node $x$ is red
   1. Color $x$ black. We are done.

Case 1: Node $x$ is black and its sibling $w$ is red
   1. Color $w$ black
   2. Color $x.p$ red
   3. Rotate $x.p$
      a. If $x$ is the left child do a left rotation
      b. If $x$ is the right child do a right rotation
   4. Now we have to change $w$
      a. If $x$ is the left child set $w = x.p.right$
      b. If $x$ is the right child set $w = x.p.left$
   5. With $x$ and our new $w$, decide on case 2, 3, or 4 from here.

Case 2: Node $x$ is black and its sibling $w$ is black and both of $w$'s children are black
   1. Color $w$ red
   2. Set $x = x.p$
      a. If our new $x$ is red, color $x$ black. We are done.
      b. If our new $x$ is black, decide on case 1, 2, 3, or 4 from here. Note that we have a new $w$ now.

Case 3: Node $x$ is black and its sibling $w$ is black and
   - If $x$ is the left child, $w$'s left child is red and $w$'s right child is black
   - If $x$ is the right child, $w$'s right child is red and $w$'s left child is black
   1. Color $w$'s child black
      a. If $x$ is the left child, color $w.left$ black
b. If \( x \) is the right child, color \( w.right \) black

2. Color \( w \) red

3. Rotate \( w \)
   a. If \( x \) is the left child do a right rotation
   b. If \( x \) is the right child do a left rotation

4. Now we have to change \( w \)
   a. If \( x \) is the left child set \( w = x.p.right \)
   b. If \( x \) is the right child set \( w = x.p.left \)

5. Proceed to case 4.

Case 4: Node \( x \) is black and its sibling \( w \) is black and
   - If \( x \) is the left child, \( w \)'s right child is red
   - If \( x \) is the right child, \( w \)'s left child is red

1. Color \( w \) the same color as \( x.p \)
2. Color \( x.p \) black

3. Color \( w \)'s child black
   a. If \( x \) is the left child, color \( w.right \) black
   b. If \( x \) is the right child, color \( w.left \) black

4. Rotate \( x.p \)
   a. If \( x \) is the left child do a left rotation
   b. If \( x \) is the right child do a right rotation

5. We are done.