EXAM

Please read all instructions, including these, carefully

- There are 9 questions on the exam, with multiple parts. You have 3 hours to work on the exam.
- The exam is open book, open notes.
- Please write your answers in the space provided on the exam and clearly mark your solutions.
- You may use the backs of the exam pages as scratch paper, or use additional pages (available at the front of the room).
- Each problem has a straightforward solution. Solutions will be graded on correctness and clarity. Partial solutions will be given partial credit.

NAME: ____________________________________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Max points</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
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<tr>
<td>2</td>
<td>15</td>
<td></td>
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<tr>
<td>3</td>
<td>10</td>
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<td>4</td>
<td>10</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>10</td>
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<td>7</td>
<td>10</td>
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<td>8</td>
<td>10</td>
<td></td>
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<tr>
<td>9</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. (a) Using the grammar below, draw a derivation tree for the following string:

\[
((\text{id} \cdot \text{id}) \text{id} (\text{id}) (()))
\]

\[
\begin{align*}
S & \rightarrow E \\
E & \rightarrow \text{id} \\
& \quad | (E \cdot E) \\
& \quad | (L) \\
L & \rightarrow LE \\
& \quad | E \\
\end{align*}
\]
(b) Give a rightmost canonical derivation for the string given in part (a).

S
E
(L)
(L E)
(L (L))
(L (E))
(L (()))
(L (E ()))
(L (L) ()))
(L (E) ()))
(L (id) ()))
(L (E (id)) ()))
(L id (id) ()))
(E id (id) ()))
((E E) id (id) ()))
((E id) id (id) ()))
((id id) id (id) ()))
2. Consider the following grammar:

1. \( S \rightarrow ABA \)
2. \( A \rightarrow Bc \)
3. \( | \ dA \)
4. \( | \ \varepsilon \)
5. \( B \rightarrow eA \)

(a) Fill in the table below with the \textsc{First} and \textsc{Follow} sets for the non-terminals in this grammar:

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Follow</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>( d, e )</td>
<td>$</td>
</tr>
<tr>
<td>A</td>
<td>( d, e, \varepsilon )</td>
<td>( c, d, e, $ )</td>
</tr>
<tr>
<td>B</td>
<td>( e )</td>
<td>( c, d, e, $ )</td>
</tr>
</tbody>
</table>

(b) Fill in the LL (1) parse table for this grammar.

<table>
<thead>
<tr>
<th></th>
<th>( c )</th>
<th>( d )</th>
<th>( e )</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>4</td>
<td>3,4</td>
<td>2,4</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(b) Fill in the LL (1) parse table for this grammar.

3. Consider the following grammar for boolean expressions (words in capitals are terminals).
Show that this grammar is ambiguous.

There are two parse trees for several strings generated by this grammar. For example, the string \textit{ID AND ID OR ID} has two parse trees:

\begin{itemize}
\item \texttt{E}
\item \texttt{E OR E}
\item \texttt{E AND E ID}
\item \texttt{ID ID}
\end{itemize}

\[ E \rightarrow E \text{ OR } E \]
\[ E \rightarrow E \text{ AND } E \]
\[ E \rightarrow \text{ NOT } E \]
\[ E \rightarrow (E) \]
\[ E \rightarrow \text{ TRUE} \]
\[ E \rightarrow \text{ FALSE} \]
\[ E \rightarrow \text{ ID} \]

\textbf{(b)} Rewrite the grammar to remove the ambiguity and enforce the intended precedence order by introducing new non-terminals. Make sure that your revised grammar accepts the same language as the original.

\begin{itemize}
\item \texttt{E}
\item \texttt{E OR T}
\item \texttt{T AND F}
\item \texttt{F}
\item \texttt{TRUE}
\item \texttt{FALSE}
\item \texttt{ID}
\item \texttt{(E)}
\end{itemize}
4. Consider the following context free grammar:

\[
S \rightarrow Aa \\
\quad | bAc \\
\quad | dc \\
\quad | bda \\
A \rightarrow d
\]

(a) Is the grammar SLR(1)? Why or why not? You don't need to construct the parse table to answer.

No, because when the stack contains \(bd\) with \(a\) next in input, there is a shift-reduce conflict. That is, we reach a certain state in the DFA by shifting \(b\) and then \(d\). Since \(a\) is in \(\text{FOLLOW}(A)\), reduce by \(A \rightarrow d\) will be indicated in the parse table cell indexed by the current state and the terminal \(a\). However, because of the production \(S \rightarrow bda\), shift \(a\) will also be indicated in the same cell.

The relevant item sets:

\[
\begin{align*}
S & \rightarrow b\cdot Ac \\
S & \rightarrow b\cdot da \\
A & \rightarrow \cdot d
\end{align*}
\]

(b) Is the grammar LR(1)? Why or why not?

Yes, because the explicit lookahead will prevent the reduce move from being indicated in the cell.

The relevant item sets:

\[
\begin{align*}
\text{shift } b & \quad S \rightarrow b\cdot Ac \\
\text{shift } d & \quad S \rightarrow A\cdot c \\
\quad S & \rightarrow b\cdot da \\
\quad A & \rightarrow \cdot d, c
\end{align*}
\]

(c) Consider the following context free grammar for the same language:

\[
S \rightarrow Aa \\
\quad | bAc \\
\quad | Bc \\
\quad | bBa \\
A \rightarrow d \\
B \rightarrow d
\]

Is this grammar LR(1)? Why or why not?

Yes.
5. Let G be the following grammar:

\begin{align*}
1 & \quad S \rightarrow SX \\
2 & \quad \quad | \quad a \\
3 & \quad X \rightarrow \varepsilon \\
4 & \quad \quad | \quad + SY \\
5 & \quad \quad | \quad Yb \\
6 & \quad Y \rightarrow \varepsilon \\
7 & \quad \quad | \quad - SXc
\end{align*}

(a) Show the parse tree for the following string:

\[ a + a - ac \]
6. You may give any context-free grammar in answer to the questions in this part; i.e. it doesn’t matter if the grammar is LL (1), unambiguous, or any other subclass of all context-free grammars.

   (a) Consider the following FIRST and FOLLOW sets:

       \[
       \begin{align*}
       \text{FIRST}(S) & = \{b, \varepsilon\} \\
       \text{FIRST}(T) & = \{b, \varepsilon\} \\
       \text{FOLLOW}(S) & = \{a, \$\} \\
       \text{FOLLOW}(T) & = \{a, b, \$\}
       \end{align*}
       \]

   (b) Give the simplest grammar (fewest productions and shortest right-hand sides) that produces these sets. \(S\) is the start symbol.

       \[
       \begin{align*}
       S & \rightarrow Sa \mid T \\
       T & \rightarrow Tb \mid \varepsilon
       \end{align*}
       \]

   (c) Give a simple example of a grammar with a shift-reduce conflict.

       \[
       \begin{align*}
       S & \rightarrow SaT \mid T \\
       T & \rightarrow TaS \mid \varepsilon
       \end{align*}
       \]

       Explanation: Consider the following sets of items:

       \[
       \begin{align*}
       \text{I}_3: & \quad S \rightarrow Sa \cdot T, a \\
       & \quad T \rightarrow \cdot TaS, a \\
       & \quad T \rightarrow \cdot, a
       \end{align*}
       \]

       \[
       \begin{align*}
       \text{I}_4: & \quad \text{GOTO}(3,T) \\
       & \quad S \rightarrow SaT \cdot, a \\
       & \quad T \rightarrow T \cdot aS, a
       \end{align*}
       \]

       In the state corresponding to \(\text{I}_4\), have an indication to reduce by \(S \rightarrow SaT\) on \(a\), and also an indication to shift on \(a\) due to \(T \rightarrow T \cdot aS\).

7. For each grammar explain why, or why not, the grammar is LL (1):

   (a) \(S \rightarrow 0 \mid 12 \mid 345\)

       **LL(1).** \textbf{FIRST sets for all right-hand-sides are distinct.}
(b) $S \rightarrow 0 \mid T \mid 1$

$T \rightarrow 1 \mid S_0$

Not LL(1). \textsc{First} sets for all right-hand-sides for any given non-terminal are not distinct ($\textsc{First}(S) = \{0, 1\}$ so $T$'s right-hand-sides $1$ and $S_0$ both have $1$ in their \textsc{First} set).

(c) $S \rightarrow 0 \mid 11 \mid 01$

Not LL(1). \textsc{First} sets for all right-hand-sides are not distinct ($0$ and $01$ have the same first set).

8. Let synthesized attribute $F.val$ give the value of the binary fraction generated by $F$ in the grammar that follows:

\[
F \rightarrow . \ L
\]

\[
L \rightarrow LB \mid B
\]

\[
B \rightarrow 0 \mid 1
\]

For instance, on input $\mathbf{101}$, $F.val = .625$.

(a) Using only synthesized attributes, give a translation scheme for the above grammar.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Attribute value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F \rightarrow . L$</td>
<td>$F.val = .</td>
</tr>
</tbody>
</table>
| $L \rightarrow L_1 B$ | $L.pos = L_1.pos + 1$
|                  | $L.val = L_1.val + 2^{L_1.pos} \times B.val$          |
| $L \rightarrow B$  | $L.pos = 1$
|                  | $L.val = 2^{L_1.pos} \times B.val$                      |
| $B \rightarrow 0$  | $B.val = 0$                                           |
| $B \rightarrow 1$  | $B.val = 1$                                            |
(c) Show the translation of the input string .101 by decorating its parse tree. Make sure your tree is drawn clearly.
9. Consider the following parsing automaton. All that is shown are the states, transitions, and the left-hand side of each item that has the “•” all the way to right. Complete the partial items and fill in all of the missing items (be sure to include the “•”). Note that the question requires only LR(0) items.