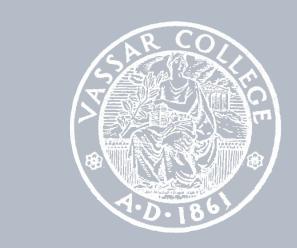
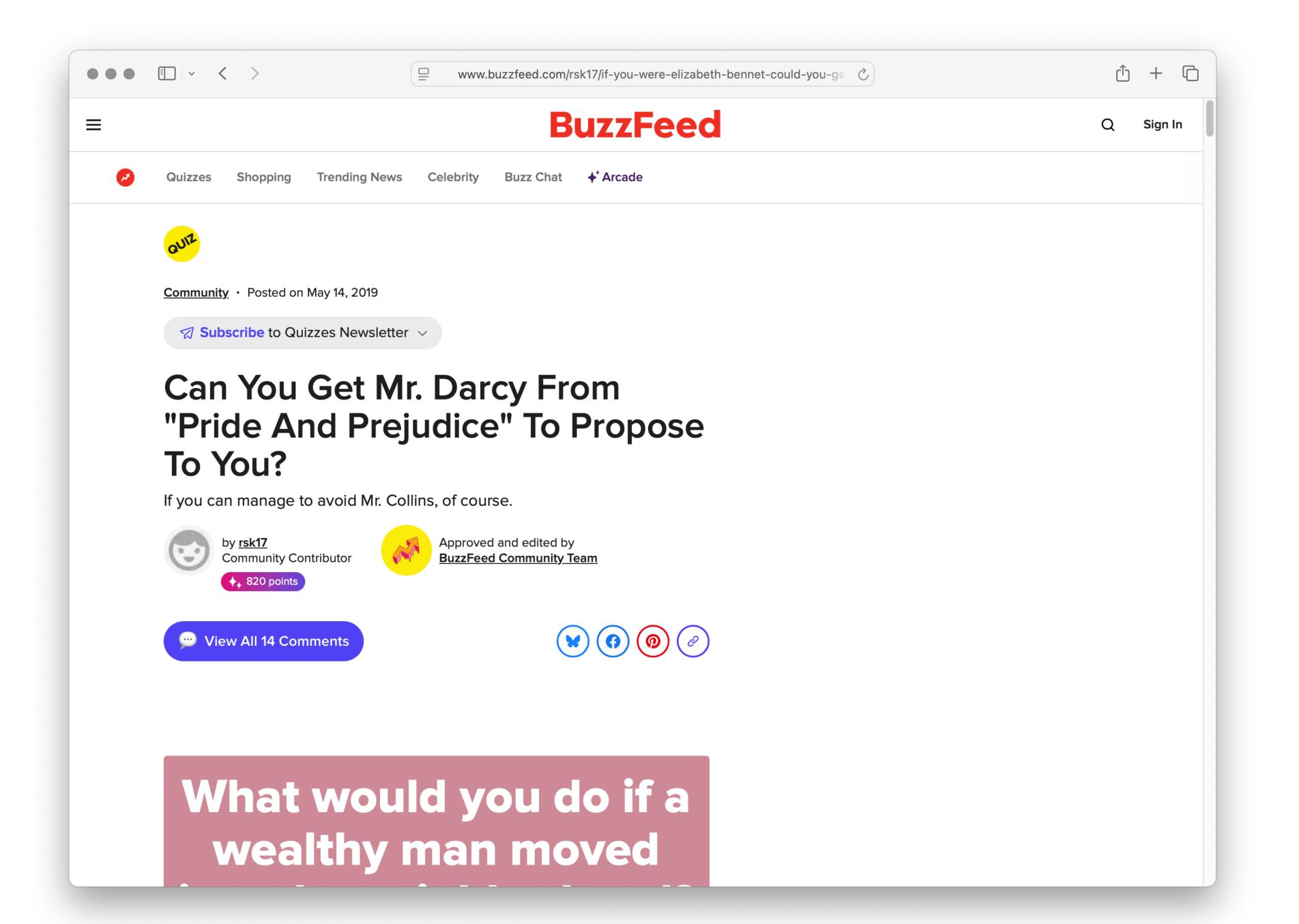
### CMPU 366 · Natural Language Processing

# Attention and Transformers

3 November 2025





### First half of the semester

### Breadth

Individual work

Learn and practice foundational NLP concepts

Rule-based systems (regular expressions)

Language models

Logistic regression

Word embeddings

Deep learning

#### First half of the semester

#### Second half of the semester

Breadth

Individual work

Learn and practice foundational NLP concepts

Rule-based systems (regular expressions)

Language models

Logistic regression

Word embeddings

Deep learning

Depth

Group work

Deep and sustained creative problem-solving to build complex projects around one topic

## Special topics

This is a chance to explore a topic we didn't cover yet. It's a good idea for this to be on the general area you'd like your final project to be in — though it doesn't need to be!

Sign up for a topic/day before the start of next class.

### Projects

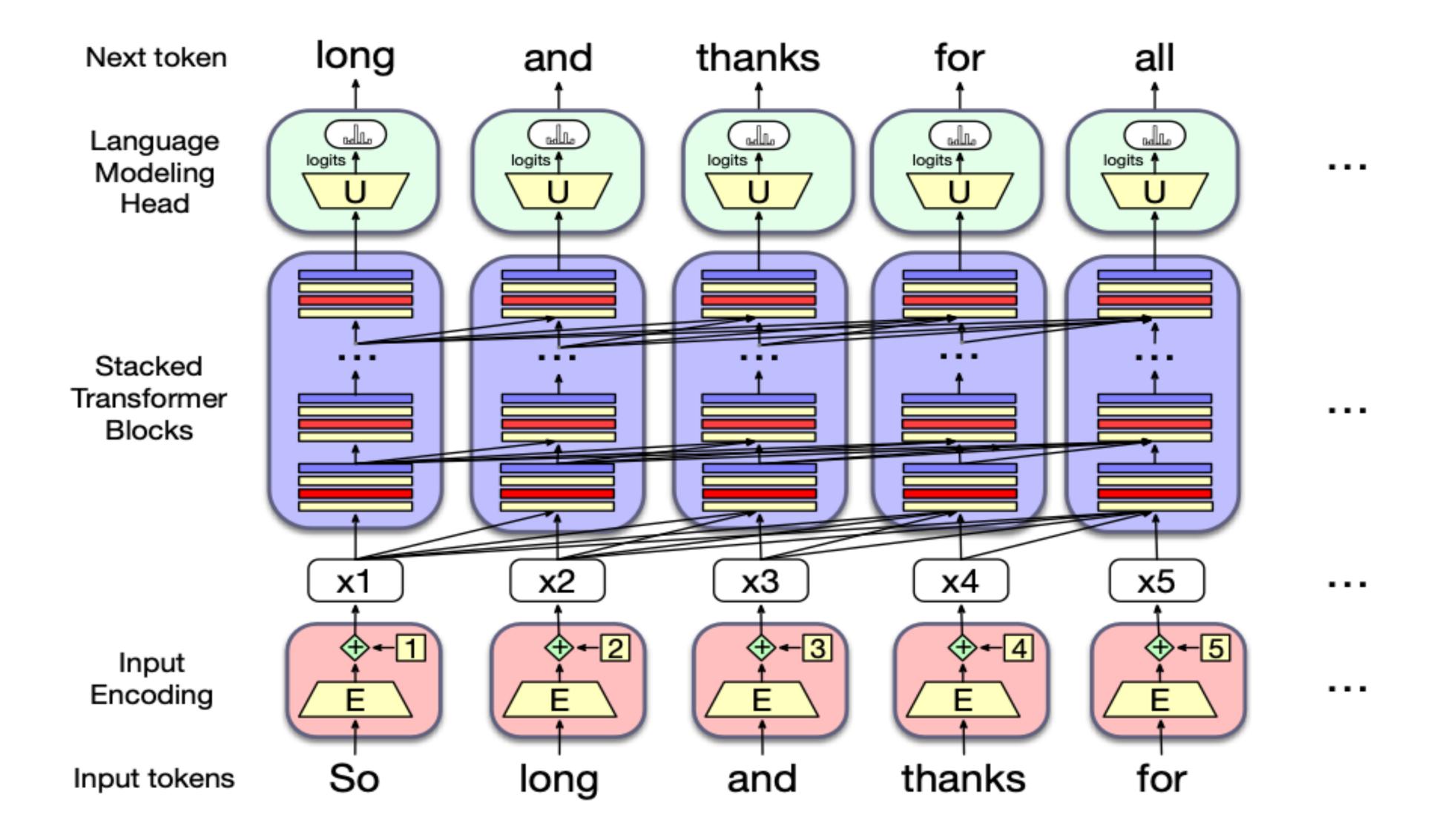
(Short) proposals will be due next Tuesday.

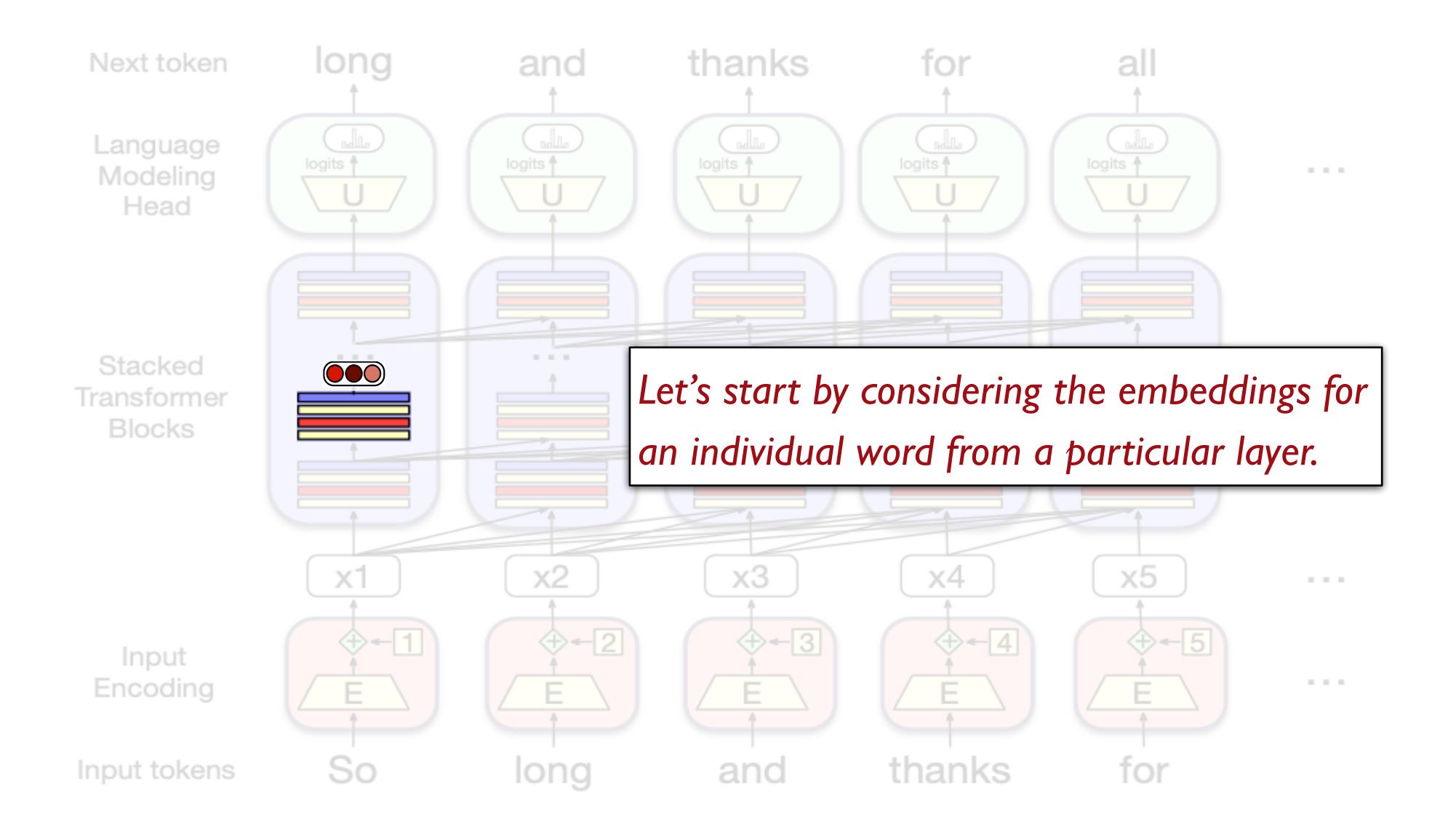
We'll use class time next Wednesday to start working on projects.

LLMs are built out of transformers.

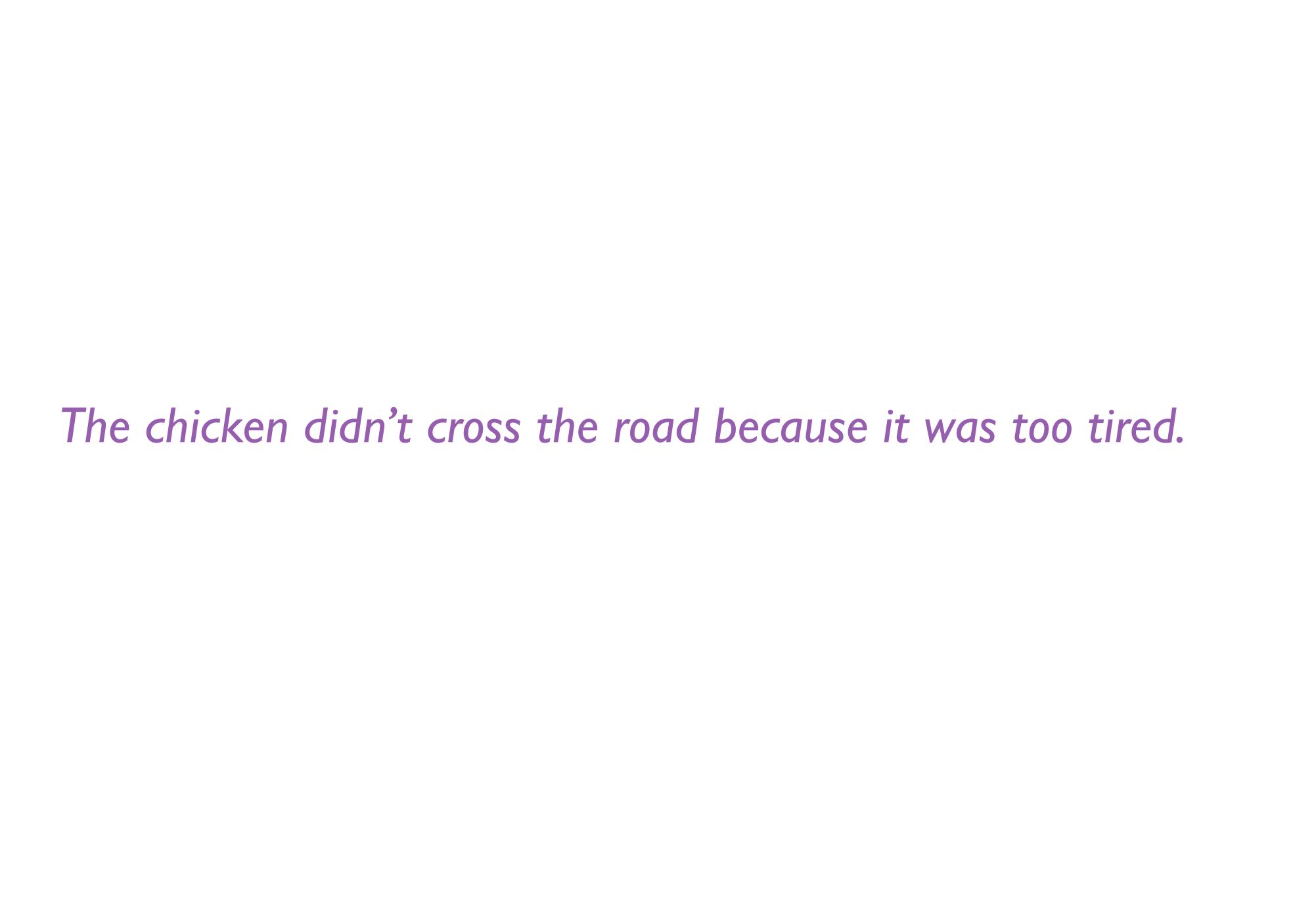
A *transformer* is a specific kind of network architecture – like a fancier feedforward network, but based on *attention*.

# Attention





The problem with embeddings like Word2vec is they're *static*; the embedding for a word doesn't reflect how its meaning changes in *context*.



What does it mean in a static embedding?

The chicken didn't cross the road because it was too tired.

A better representation of the meaning of a word would be different in different contexts!

This is called a contextual embedding.

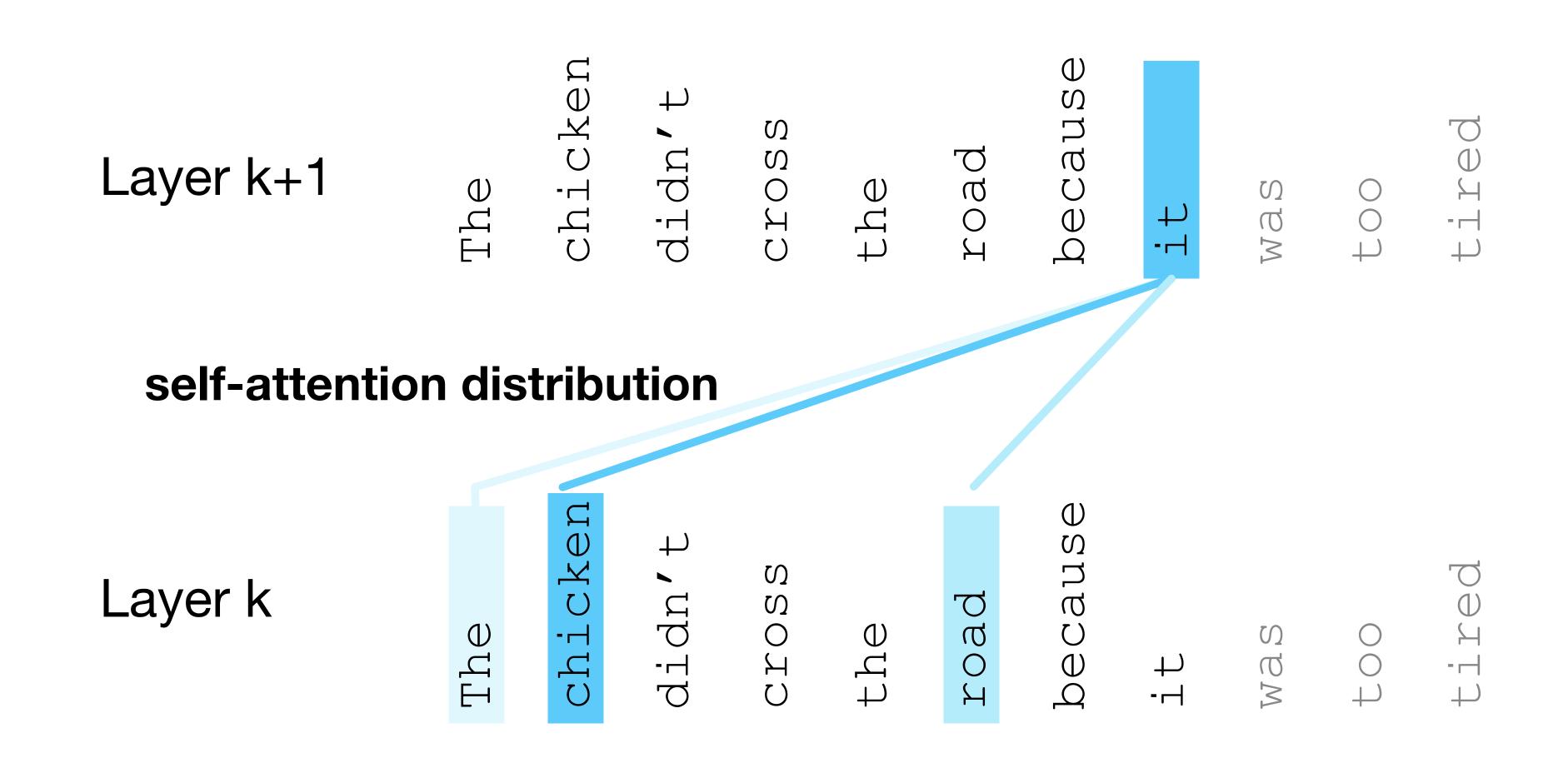
The chicken didn't cross the road because it was too tired.

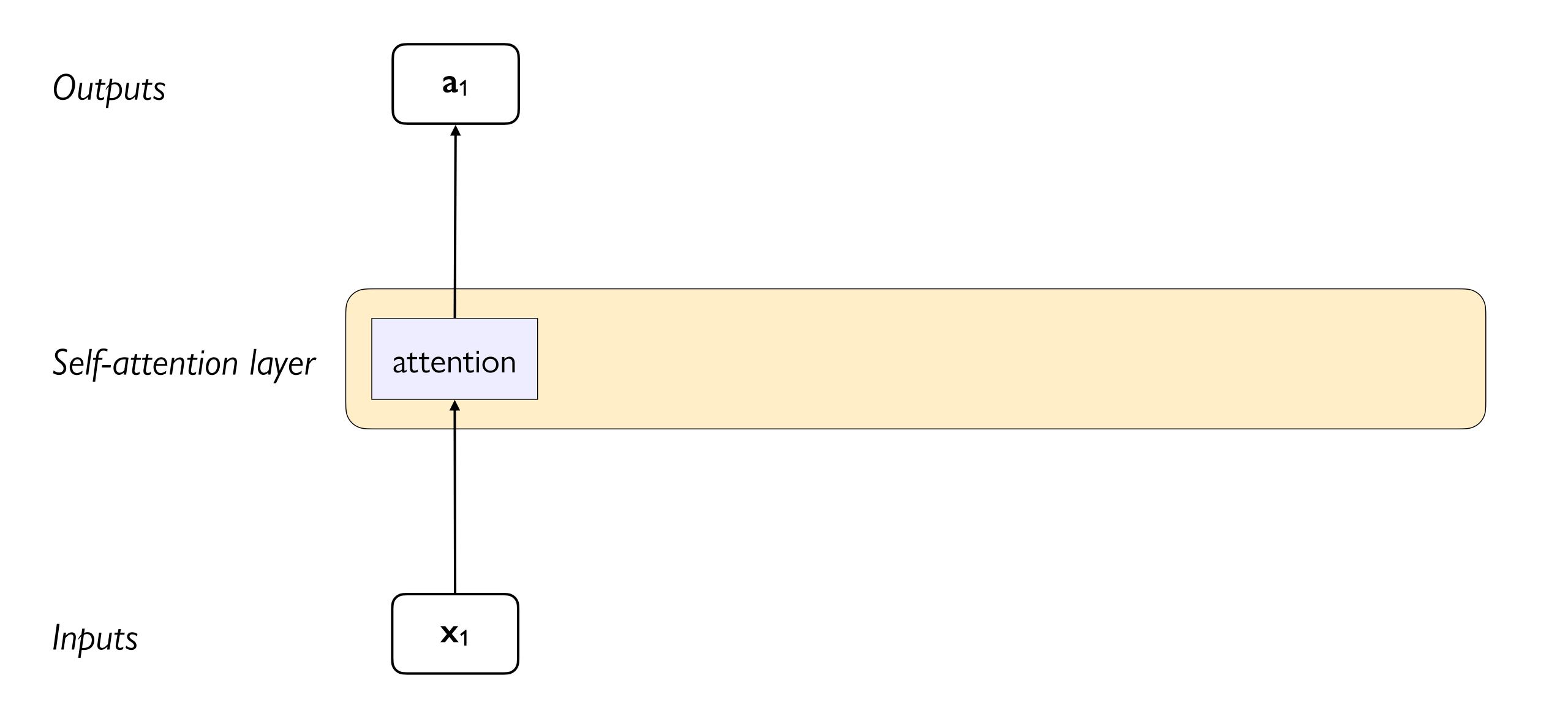
The chicken didn't cross the road because it was too wide.

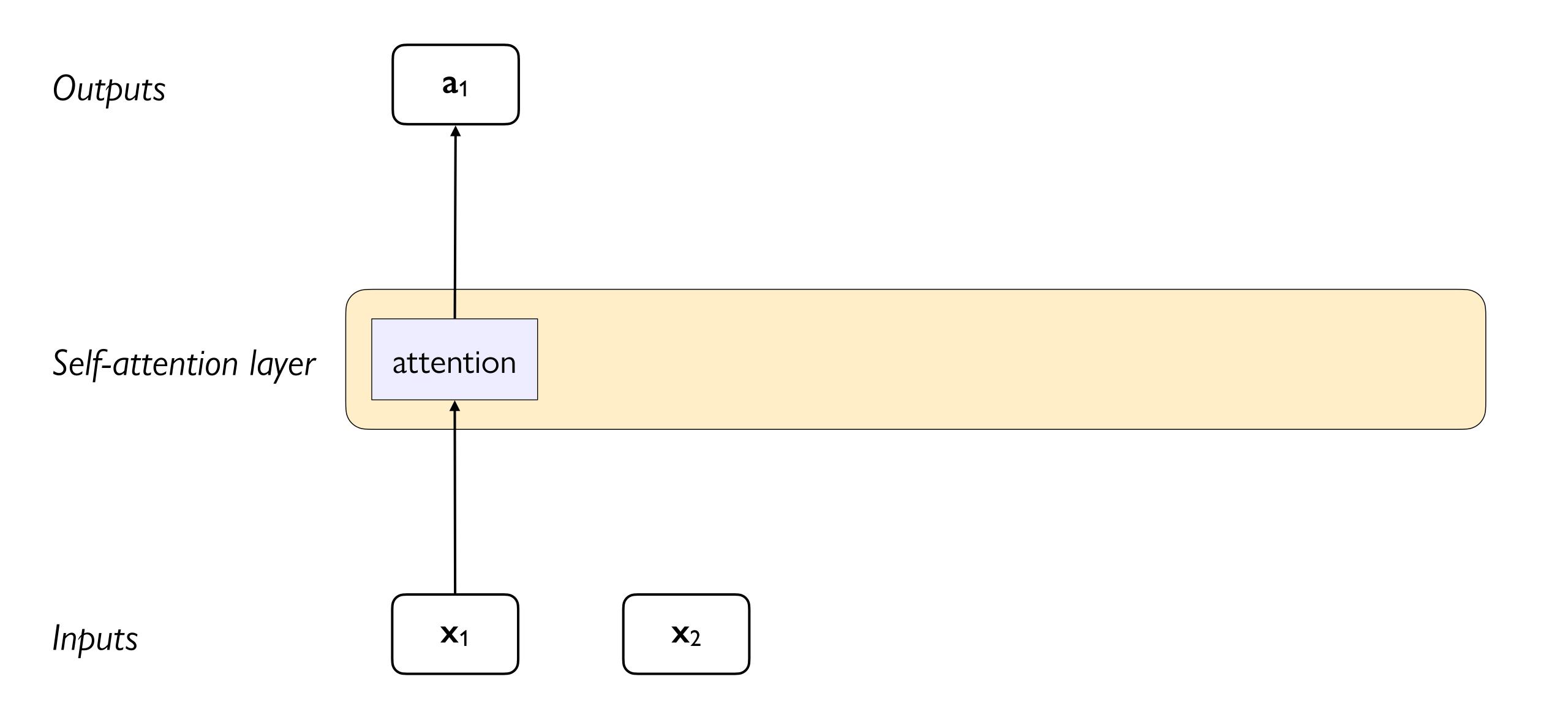
The chicken didn't cross the road because it...

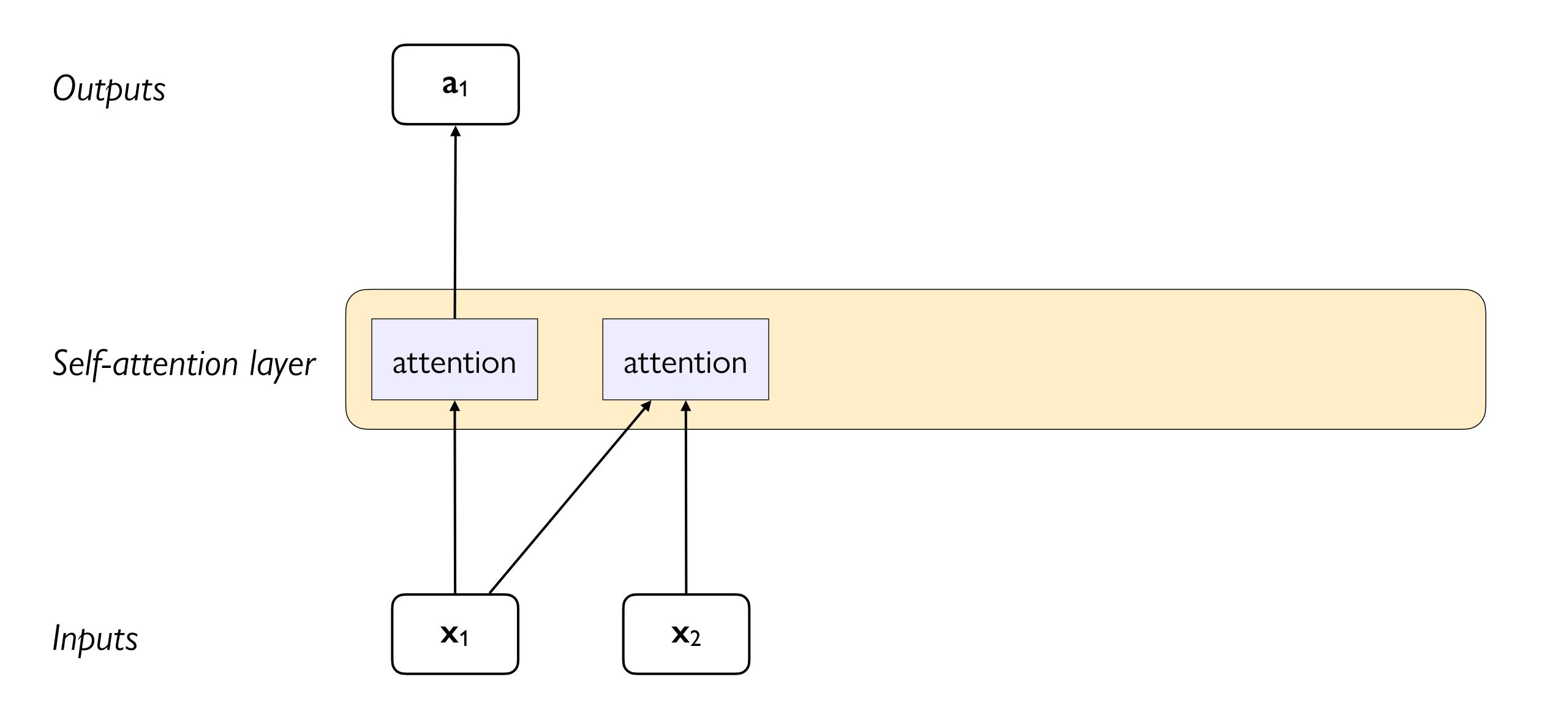
A transformer uses *attention* to build a contextual representation of a word by selectively integrating information from all the neighboring words.

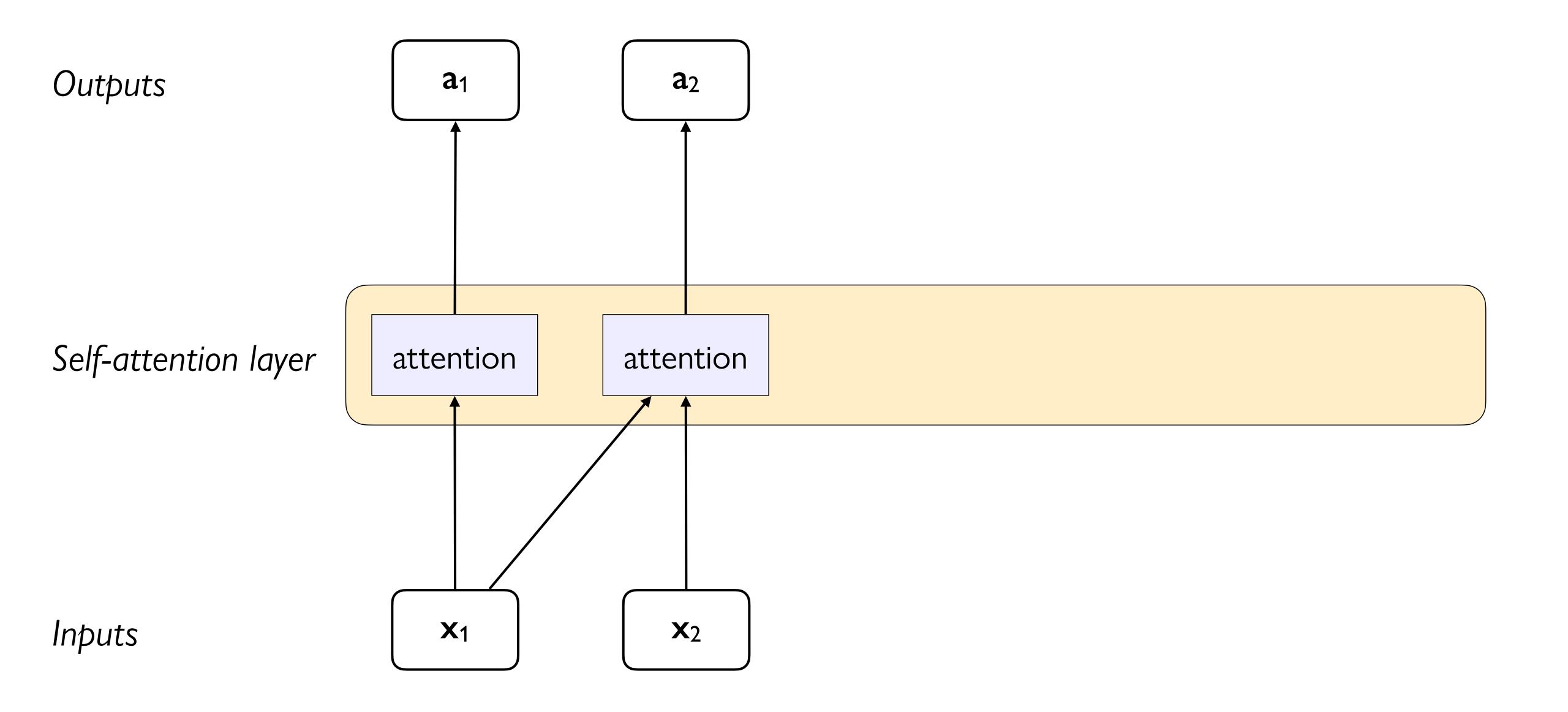
### columns corresponding to input tokens

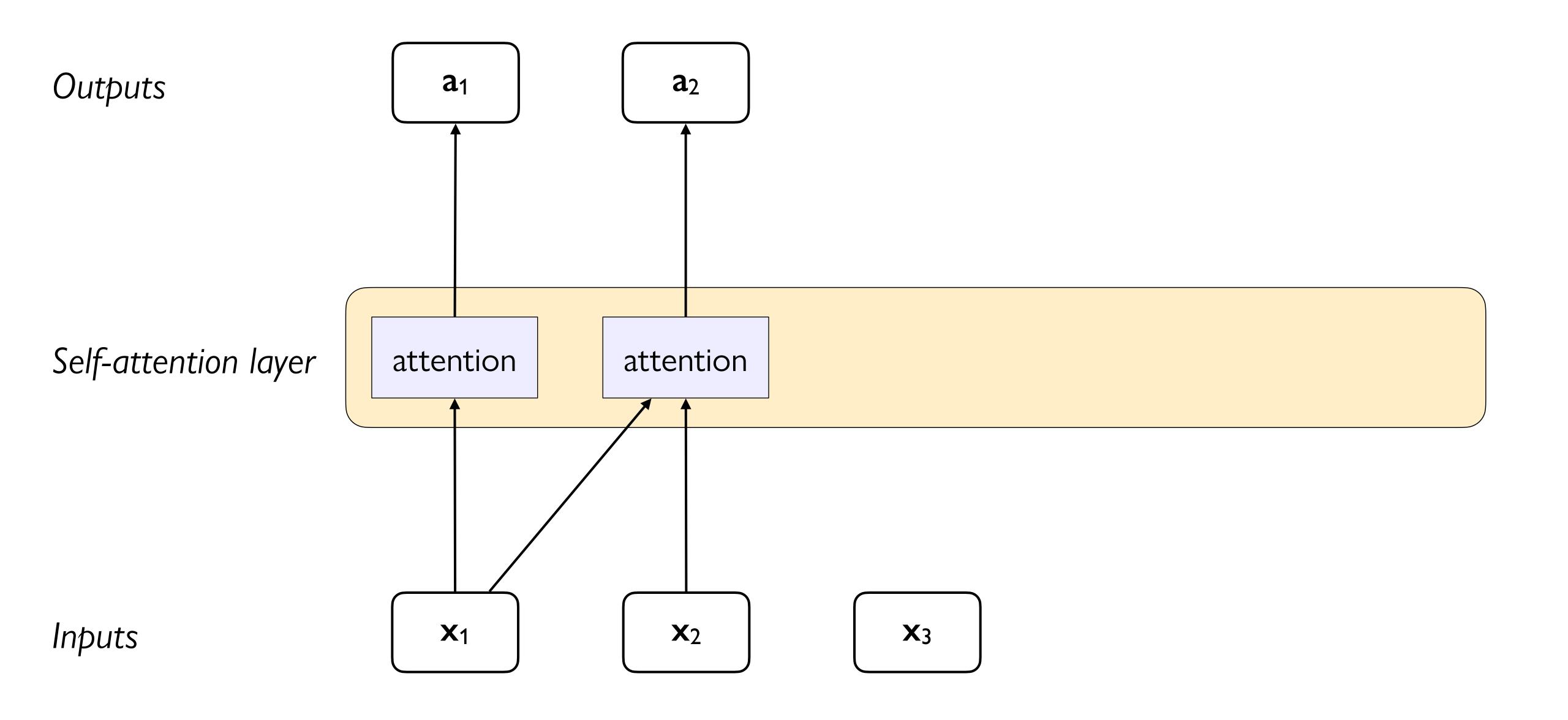


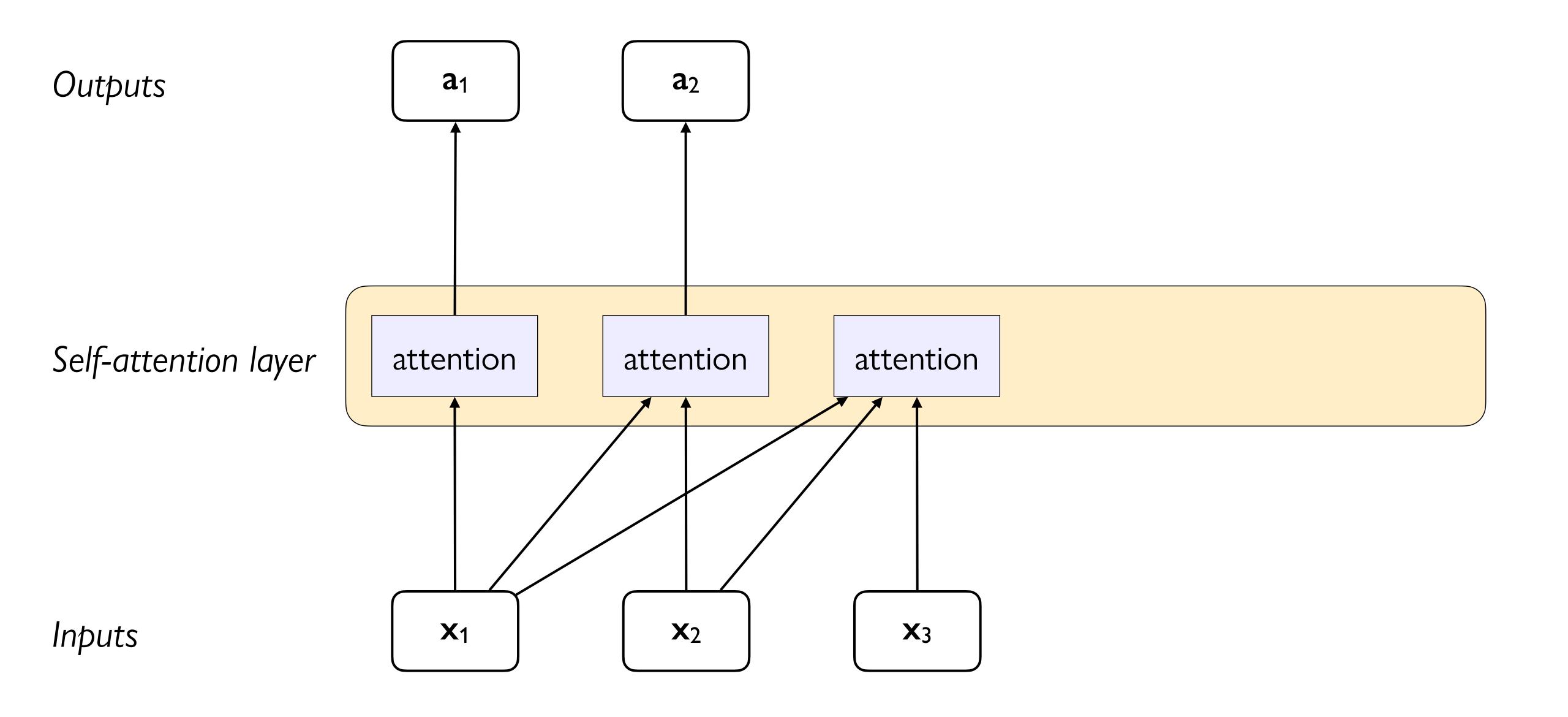


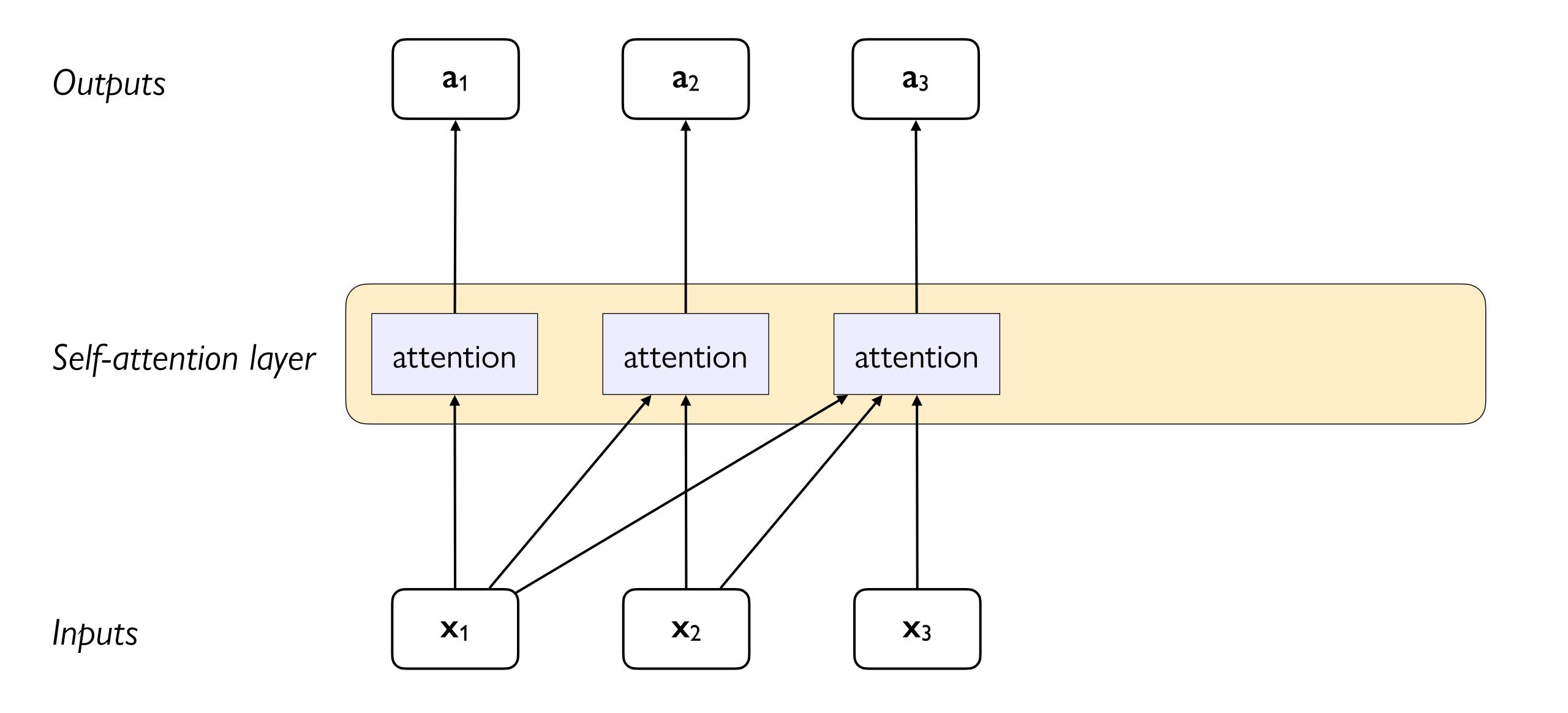


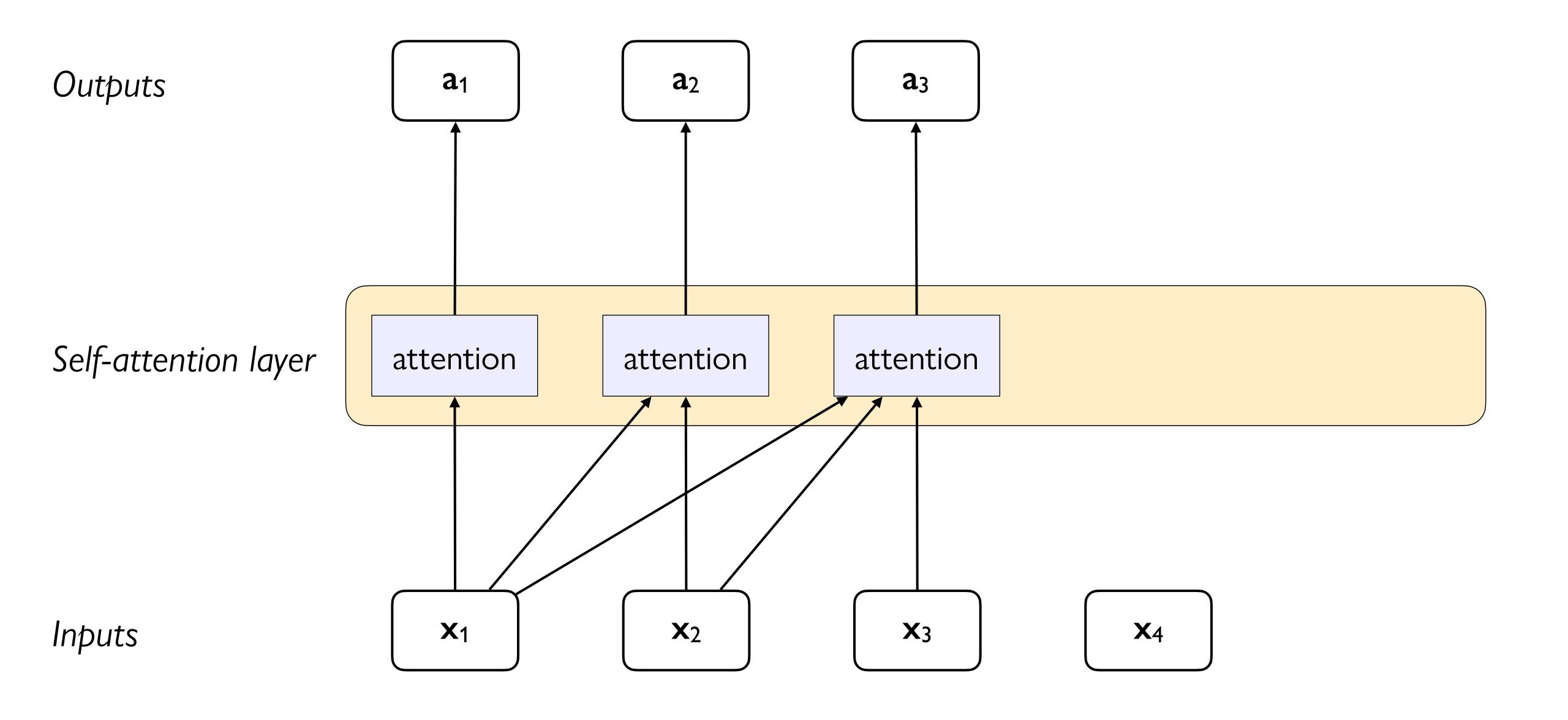


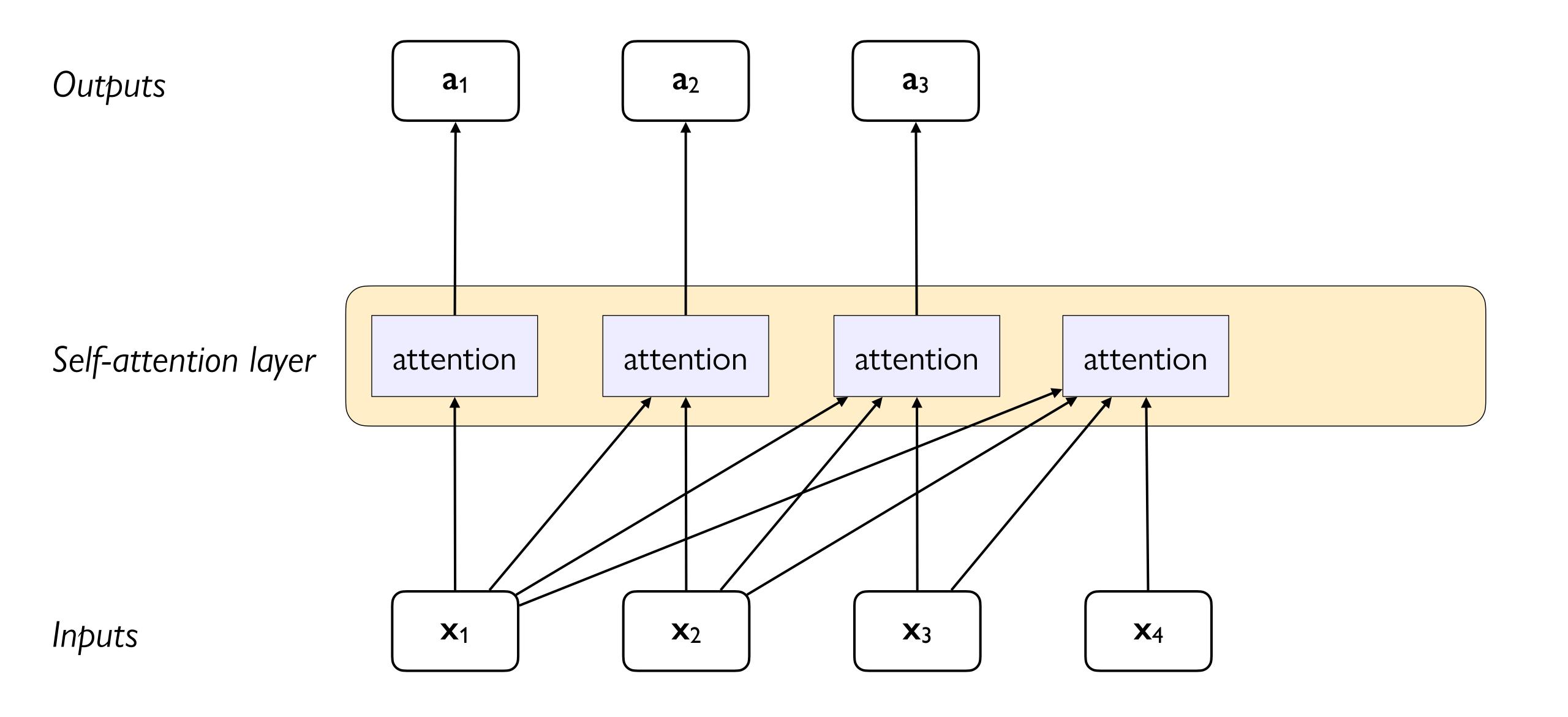


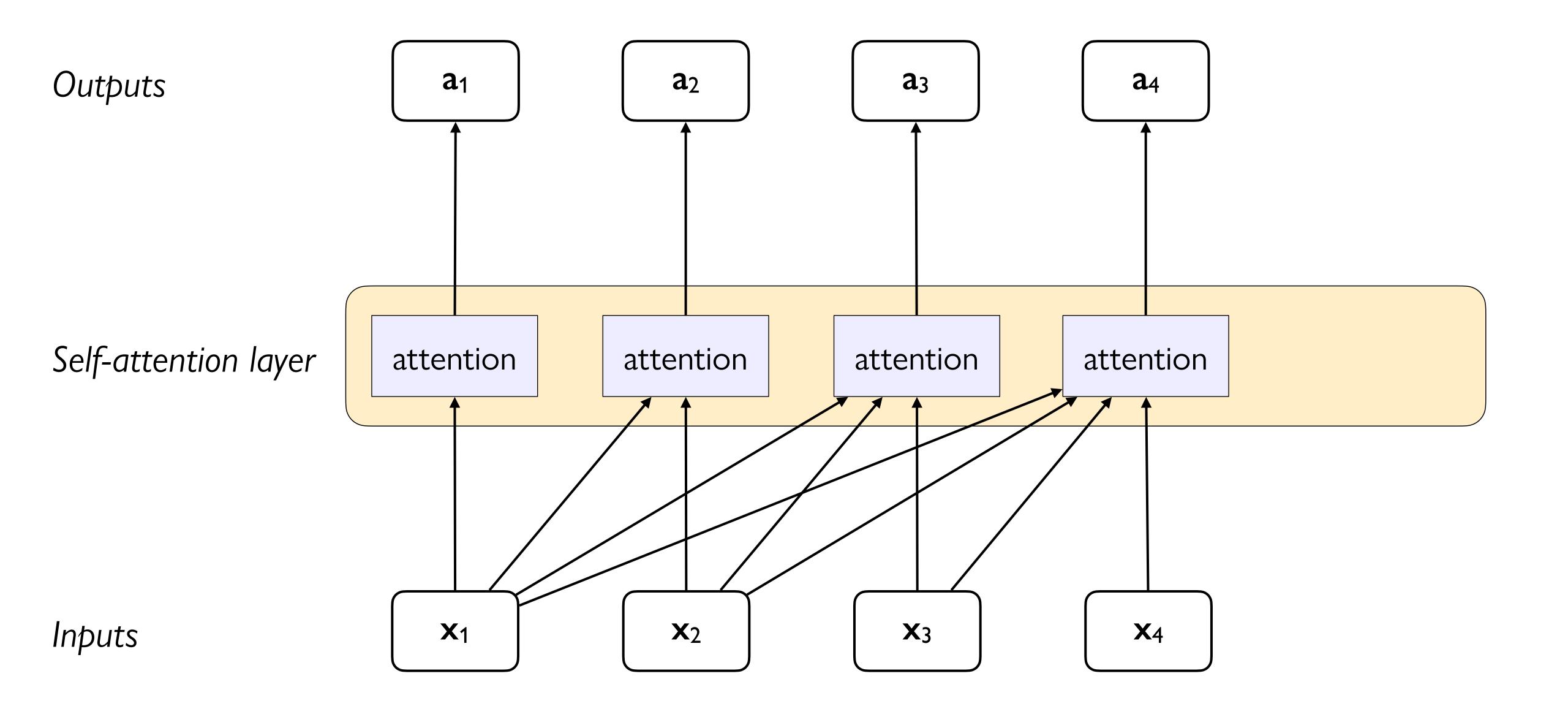


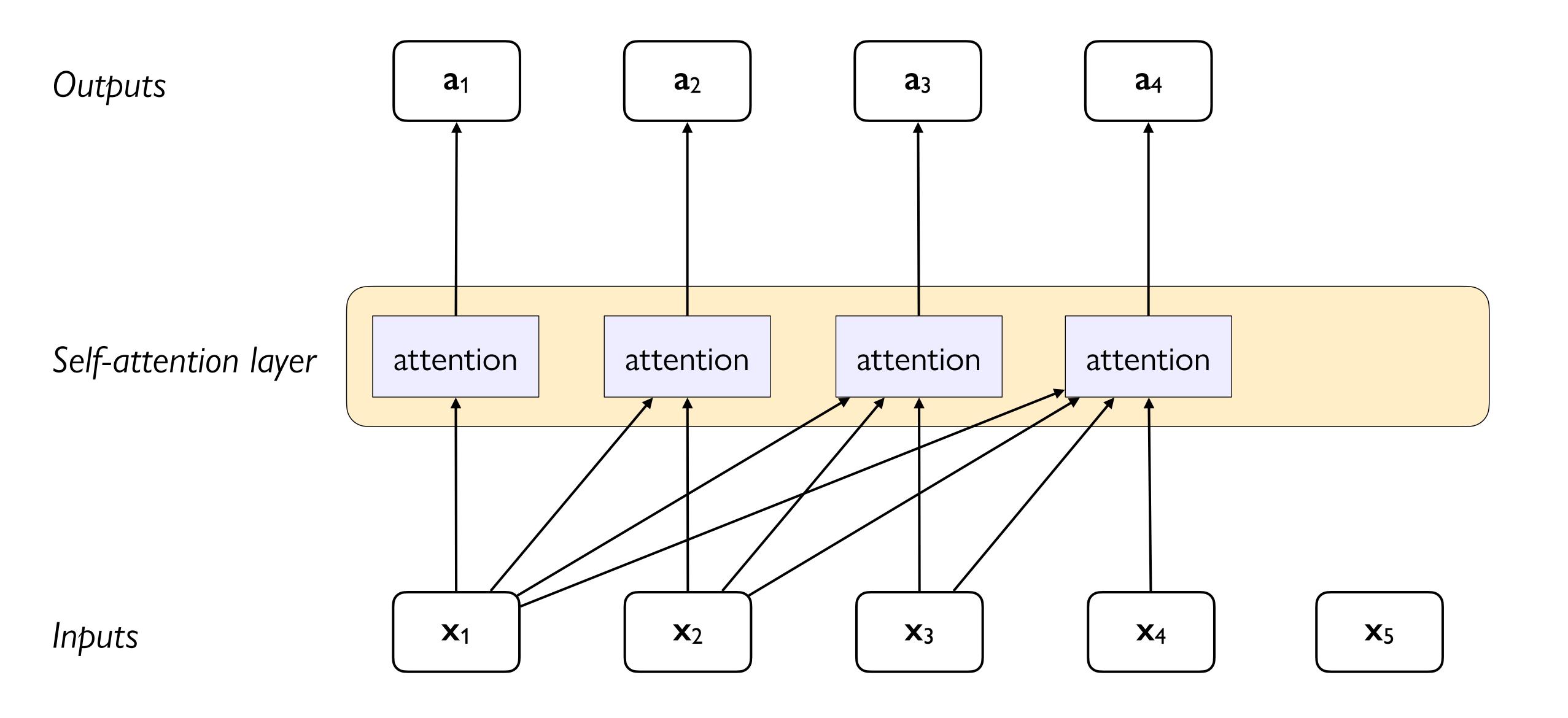


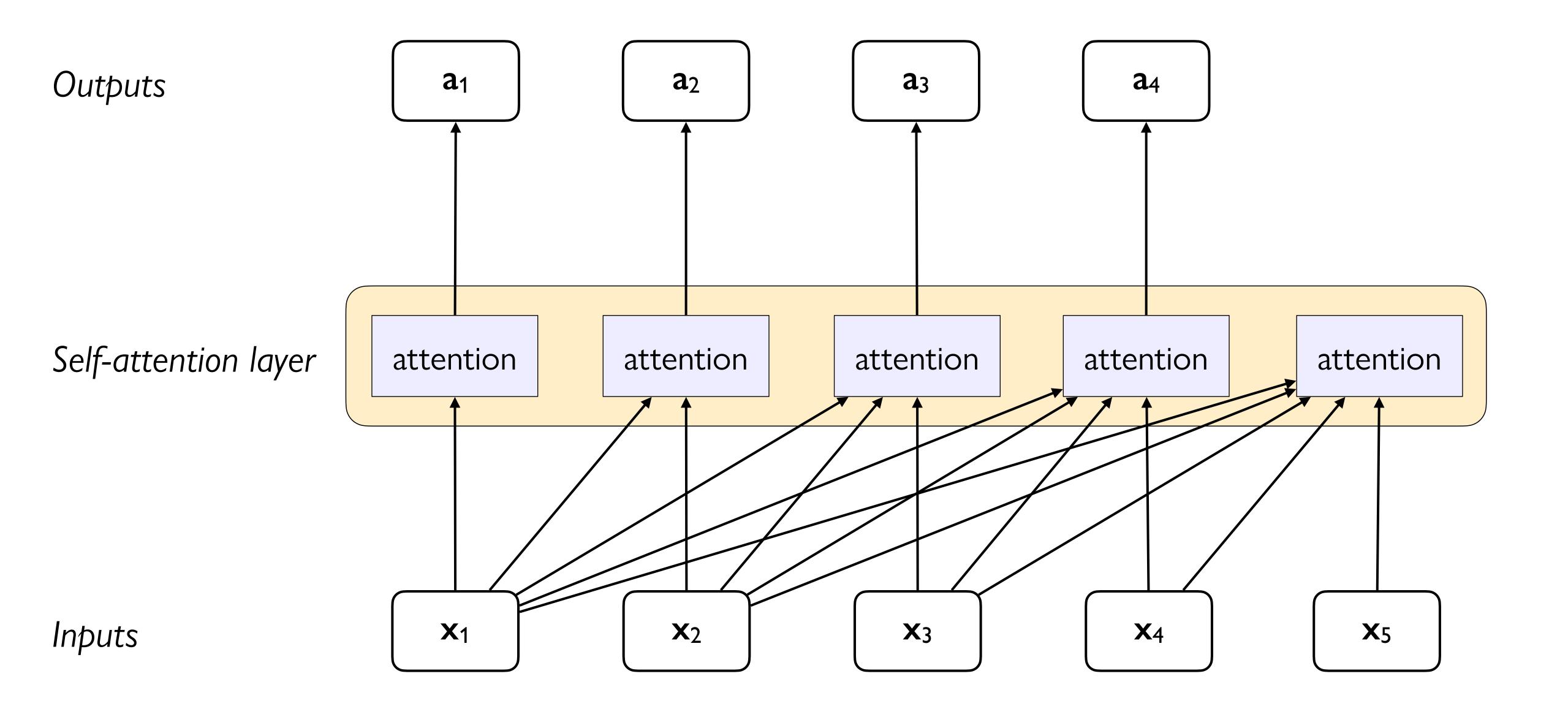


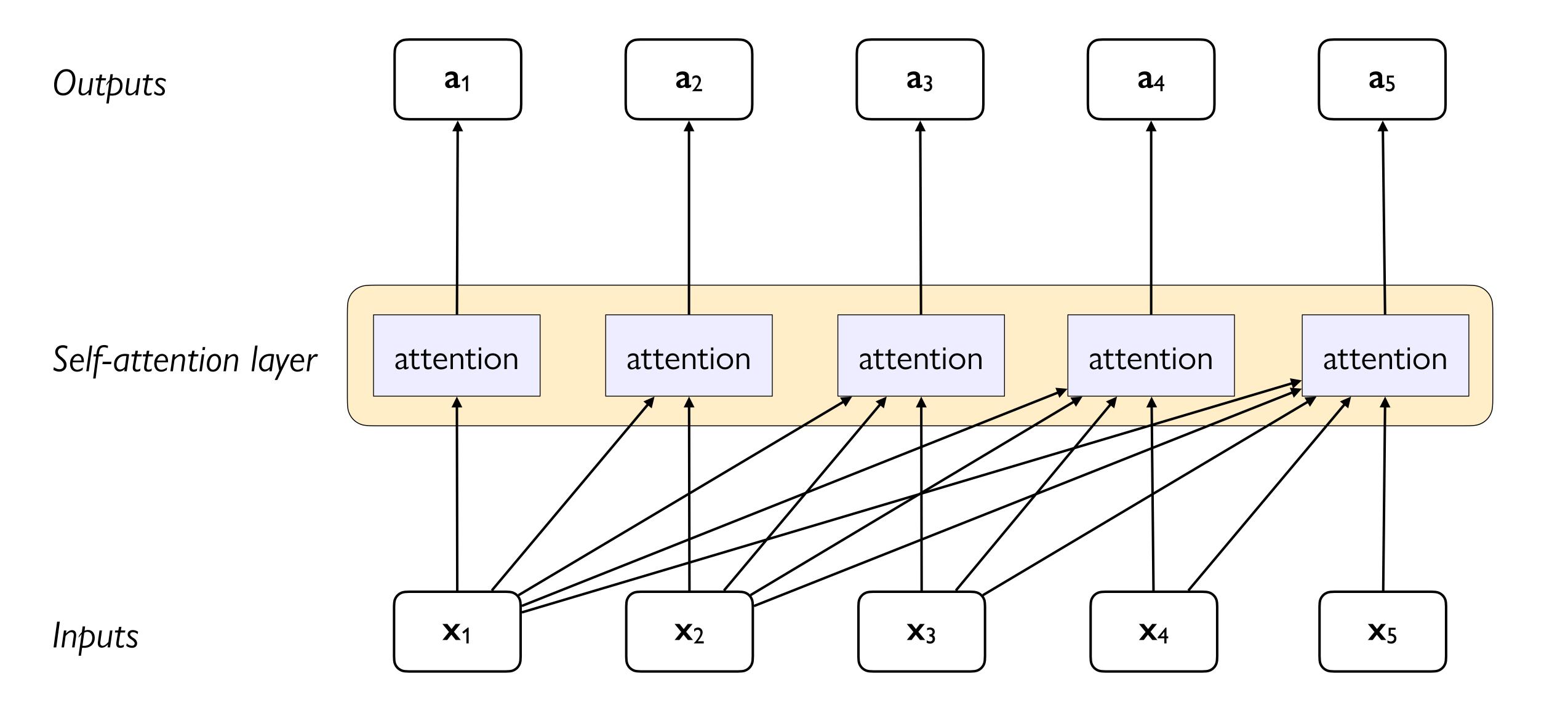












Given a sequence of token embeddings

we can produce a sum of the embeddings weighted by their similarity to  $\mathbf{x}_i$ :

$$\mathbf{a}_i = \sum_{j \le i} \alpha_{ij} \mathbf{x}_j$$

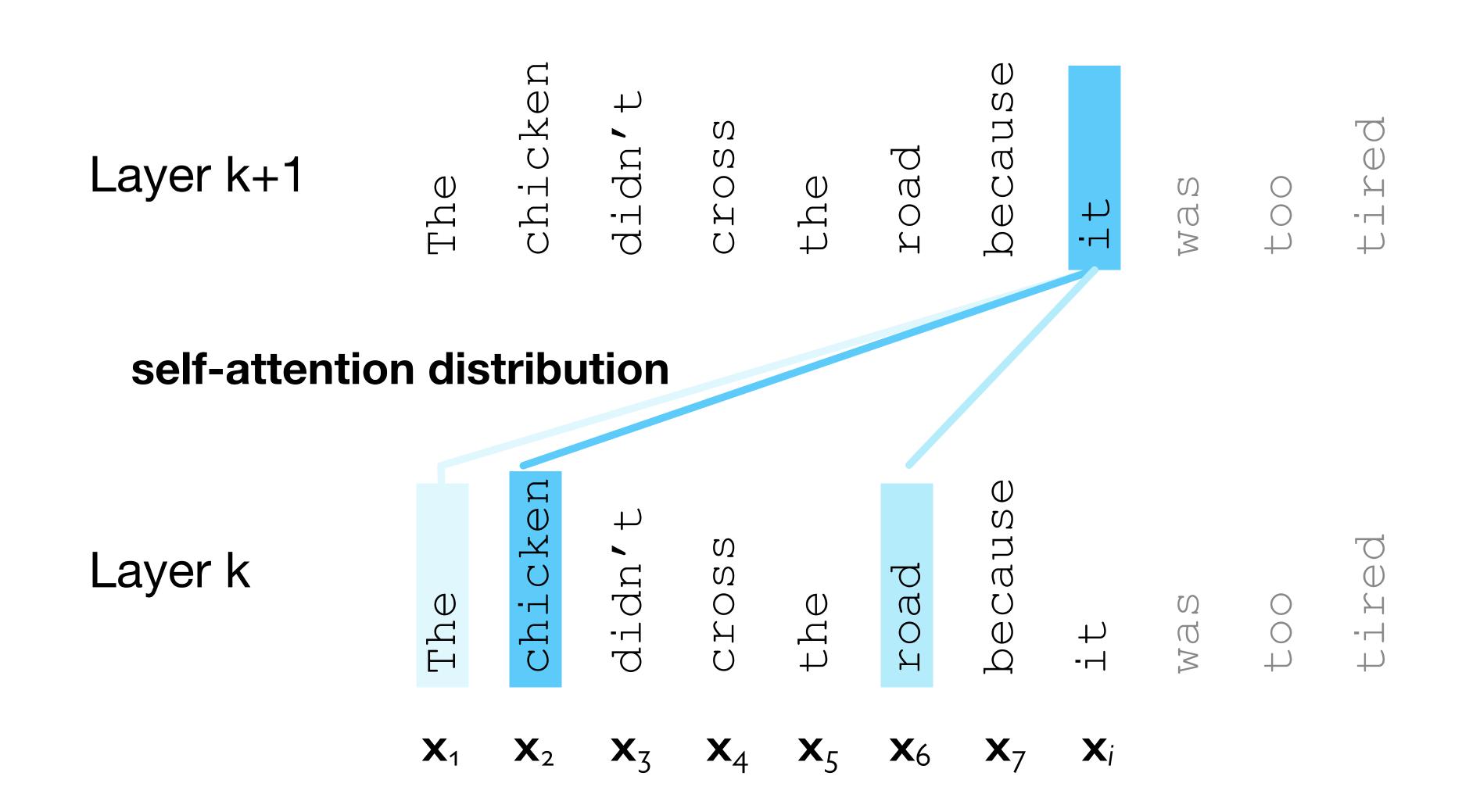
where the weight

$$\alpha_{ij} = \text{softmax}(\text{score}(\mathbf{x}_i, \mathbf{x}_j)) \ \forall j \leq i,$$

and we can simply define

$$score(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$$

### columns corresponding to input tokens



An actual attention head is slightly more complicated.

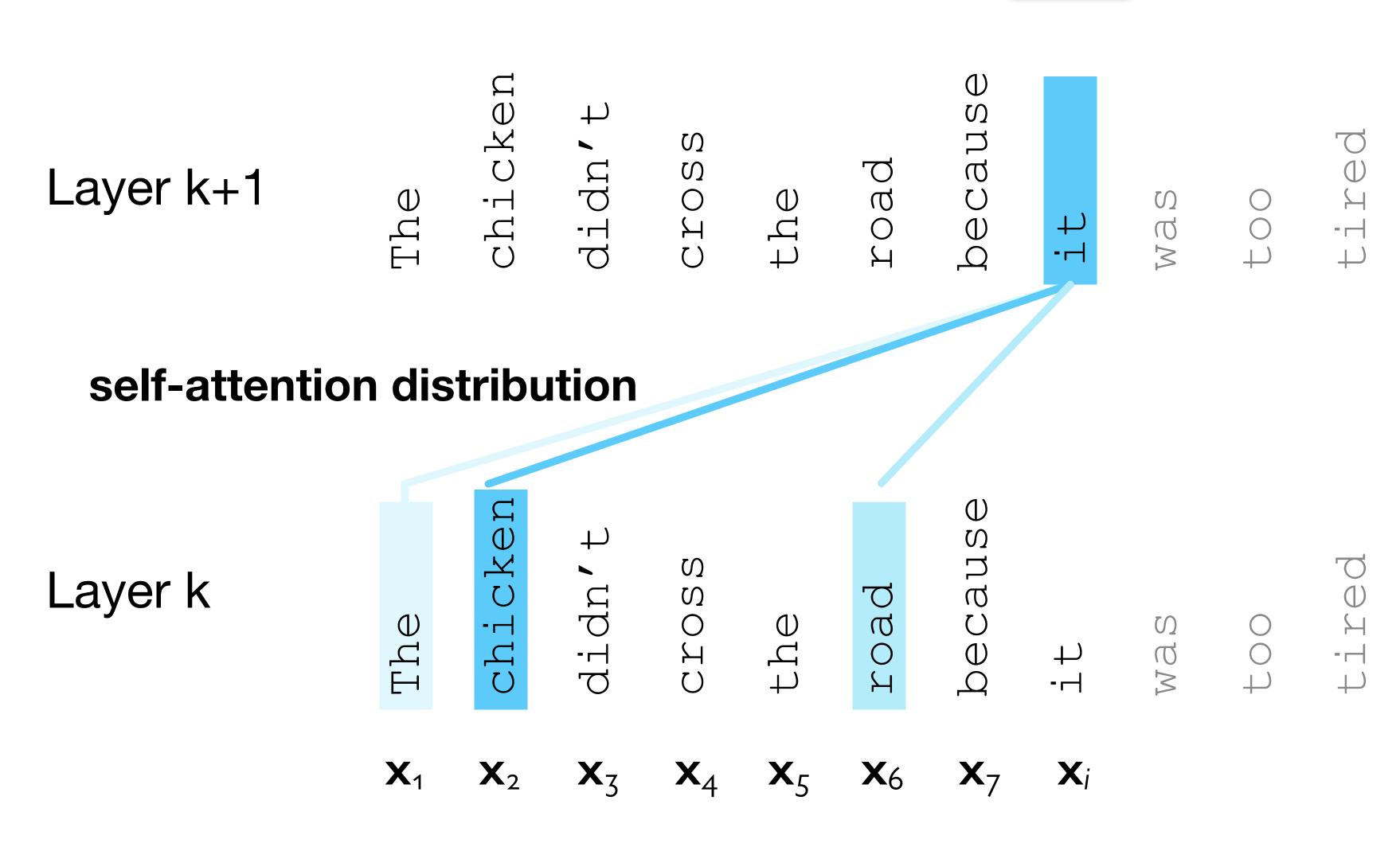
Instead of using vectors like  $\mathbf{x}_i$  and  $\mathbf{x}_4$  directly, we'll represent three separate *roles* each vector  $\mathbf{x}_i$  plays:

Query: As the current element being compared to the preceding inputs

Key: As a preceding input that is being compared to the current element to determine a similarity

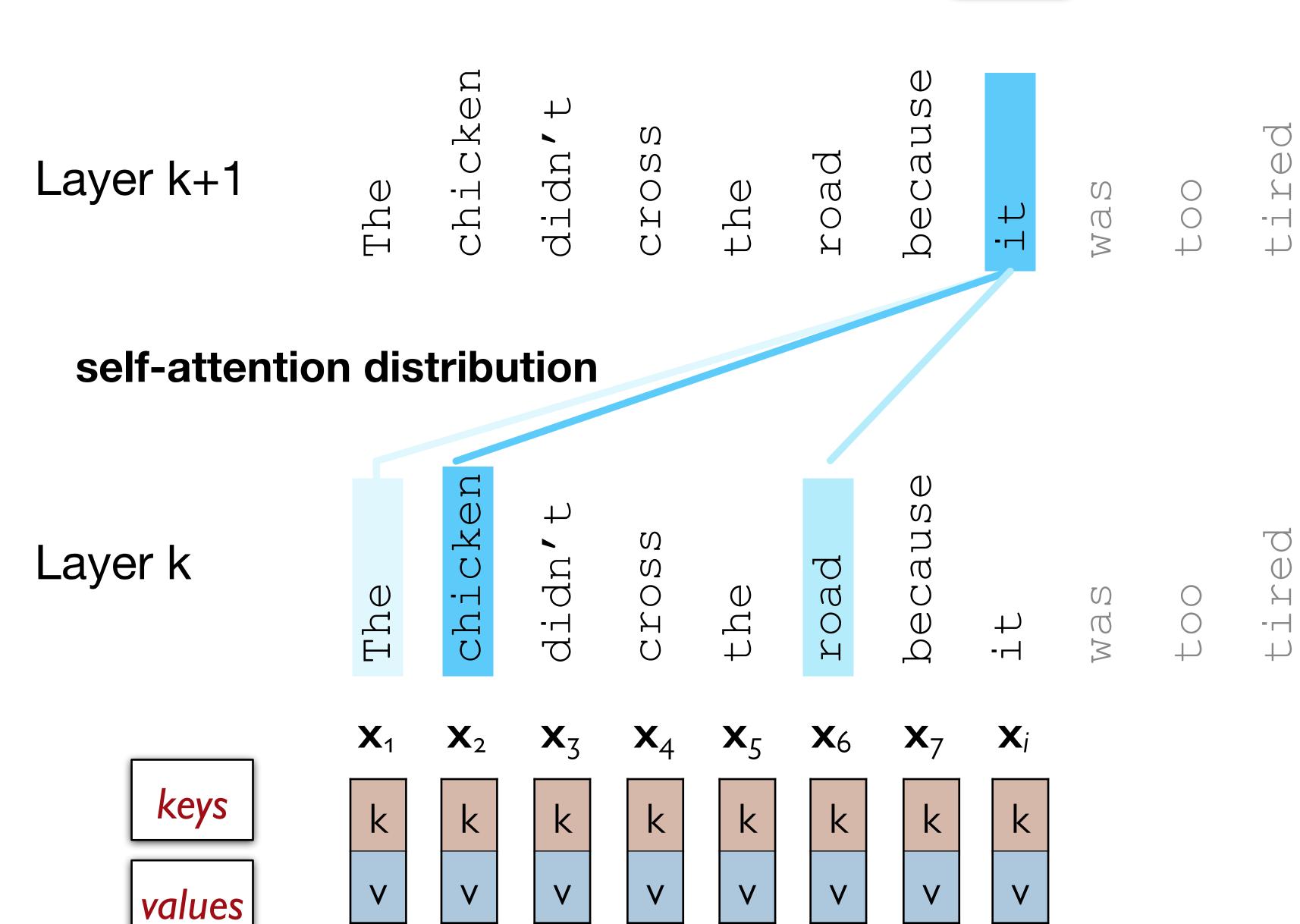
Value: a value of a preceding element that gets weighted and summed.





values





We'll use weight matrices  $\mathbf{WQ}$ ,  $\mathbf{WK}$ , and  $\mathbf{WV}$  to project each vector  $\mathbf{x}_i$  into a representation of its role as a

query: 
$$q_i = x_i WQ$$

key: 
$$\mathbf{k}_i = \mathbf{x}_i \mathbf{W}^K$$

value: 
$$\mathbf{v}_i = \mathbf{x}_i \mathbf{W}^{\mathsf{V}}$$

To compute the similarity of the current element  $\mathbf{x}_i$  with some prior element  $\mathbf{x}_j$ , we'll use the dot product between the projections  $\mathbf{q}_i$  and  $\mathbf{k}_j$ .

And instead of summing up  $\mathbf{x}_j$ , we'll sum up the projection  $\mathbf{v}_j$ .

Simplified

### De-simplified

$$\mathbf{q}_i = \mathbf{x}_i \mathbf{W}^{\mathbf{Q}}$$
 $\mathbf{k}_j = \mathbf{x}_j \mathbf{W}^{\mathbf{K}}$ 

$$\mathbf{k}_{j} = \mathbf{x}_{j} \mathbf{W}^{\mathbf{K}}$$

$$\mathbf{v}_j = \mathbf{x}_j \mathbf{W}^{\mathbf{V}}$$

$$score(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$$

$$score(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{q}_i \cdot \mathbf{k}_j$$

$$\alpha_{ij} = \text{softmax}(\text{score}(\mathbf{x}_i, \mathbf{x}_i)) \ \forall j \leq i$$

$$\alpha_{ij} = \text{softmax}(\text{score}(\mathbf{x}_i, \mathbf{x}_j)) \ \forall j \leq i$$

$$\mathbf{a}_i = \sum_{j \le i} \alpha_{ij} \mathbf{x}_j$$

$$\mathbf{a}_i = \sum_{j \le i} \alpha_{ij} \mathbf{v}_j$$

 $d_k$  is the dimensionality of the query and key vectors

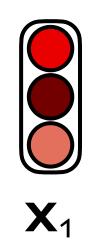
$$\mathbf{q}_i = \mathbf{x}_i \mathbf{W}^{\mathbf{Q}}$$
 $\mathbf{k}_j = \mathbf{x}_j \mathbf{W}^{\mathbf{K}}$ 
 $\mathbf{v}_j = \mathbf{v}_j \mathbf{W}^{\mathbf{V}}$ 

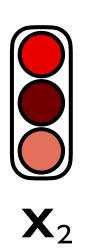
$$score(\mathbf{x}_i, \mathbf{x}_j) = \frac{\mathbf{q}_i \cdot \mathbf{k}_j}{\sqrt{d_k}} - score(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{q}_i \cdot \mathbf{k}_j$$

This is a practical change to avoid numerical issues during training.

$$\alpha_{ij} = \text{softmax}(\text{score}(\mathbf{x}_i, \mathbf{x}_j)) \ \forall j \leq i$$

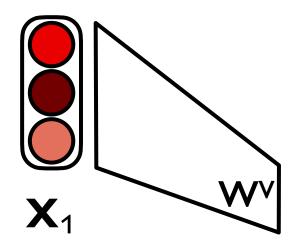
$$\mathbf{a}_i = \sum_{j \le i} \alpha_{ij} \mathbf{v}_j$$

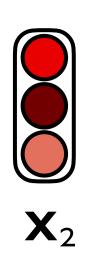


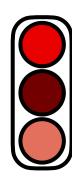




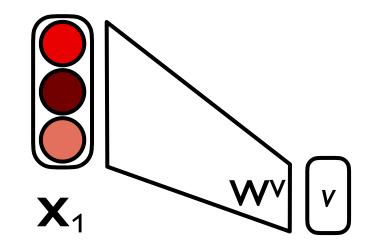
 $X_3$ 

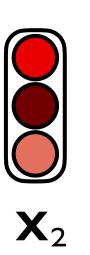


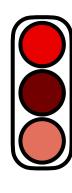




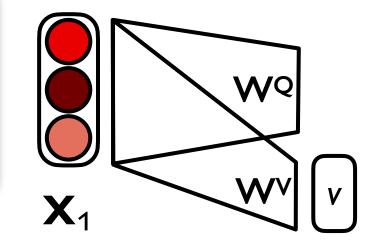
 $\mathbf{X}_3$ 

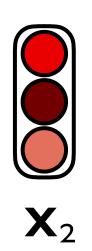


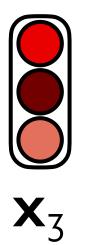


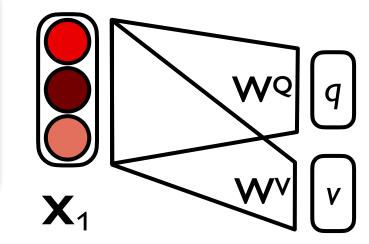


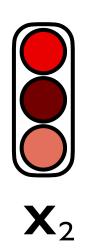
 $\mathbf{X}_3$ 

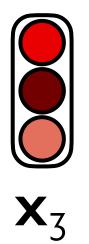


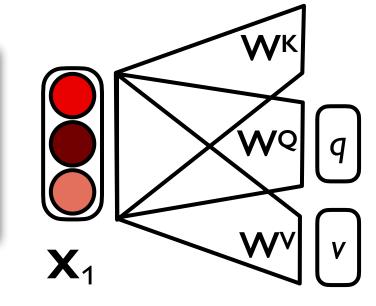


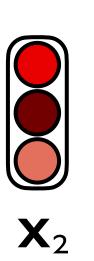


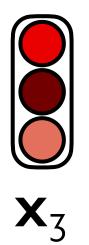


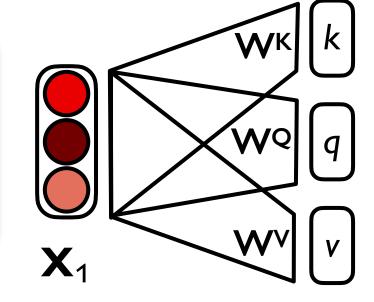


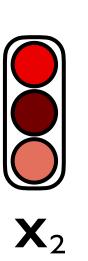


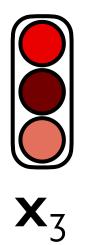


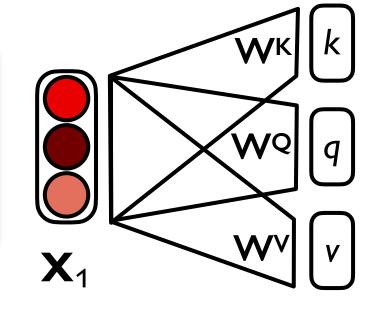


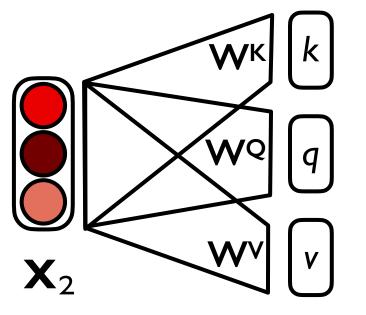


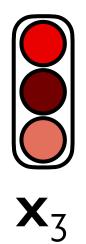


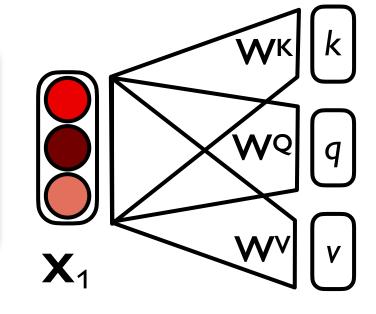


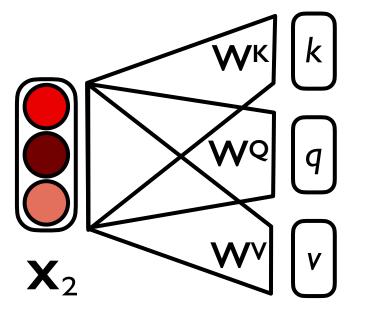


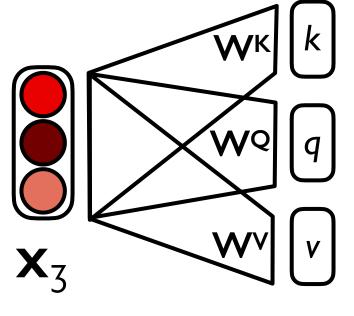


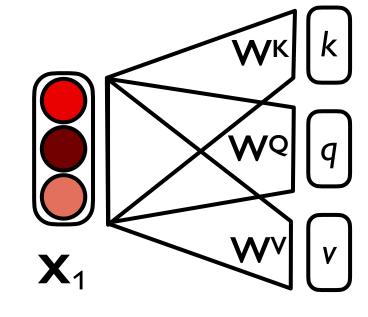


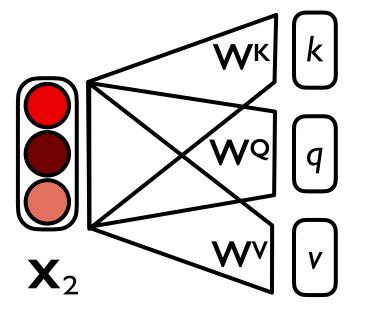


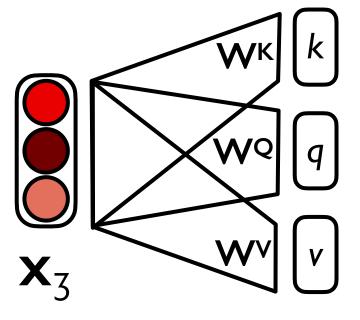


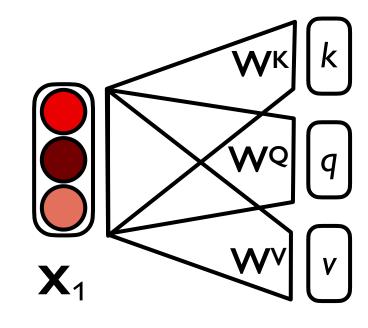


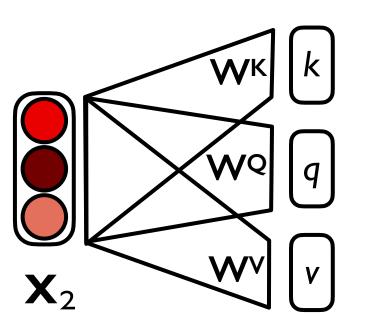


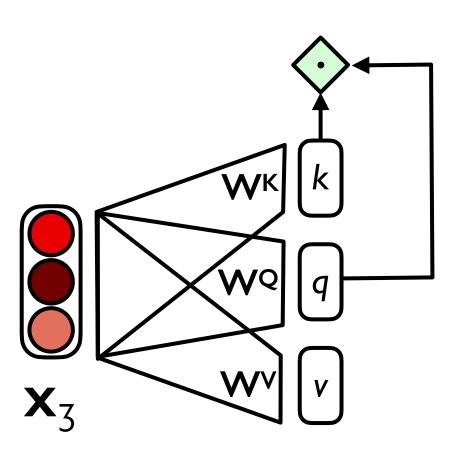


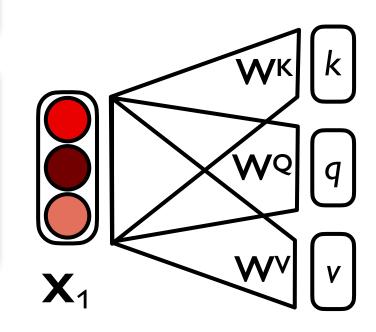


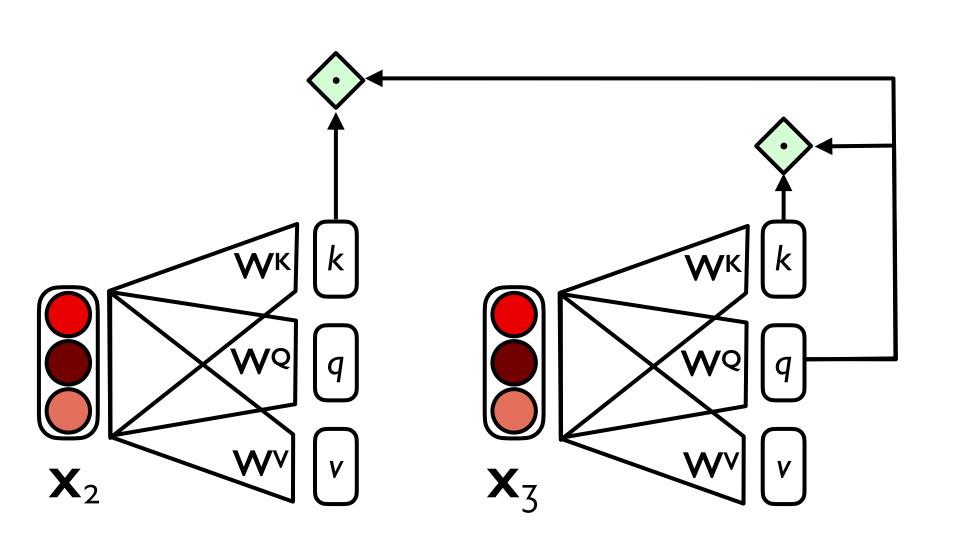


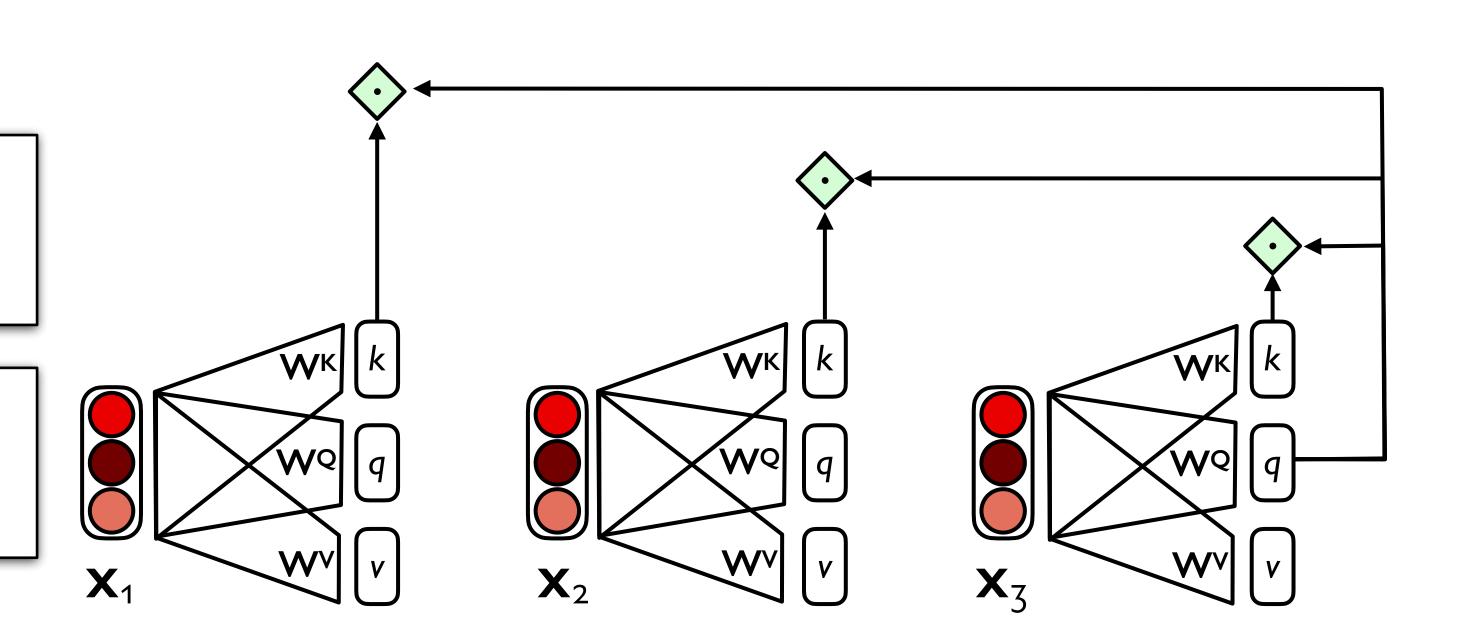






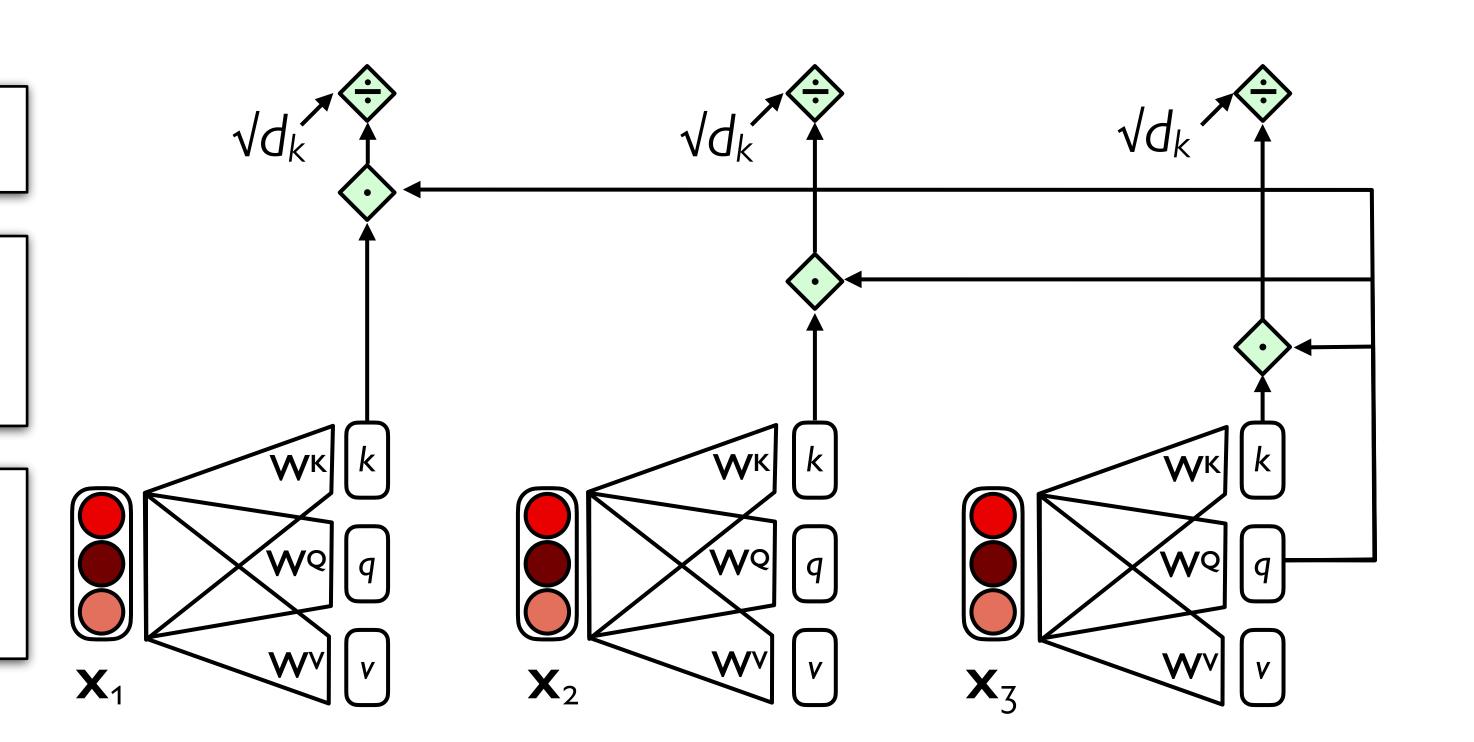






#### Divide score by √d<sub>k</sub>

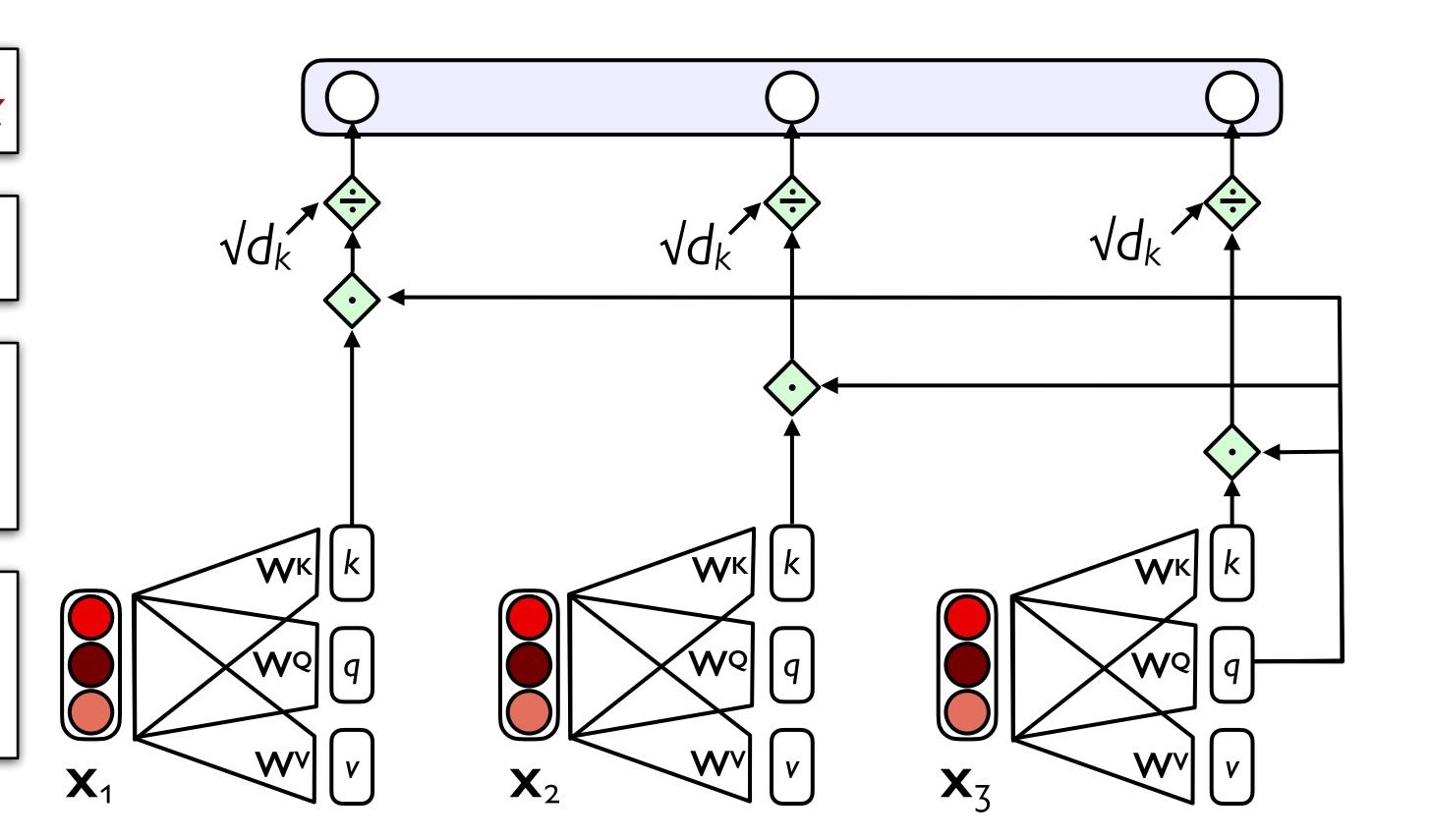
Compare  $x_3$ 's query with the keys for  $x_1$ ,  $x_2$ , and  $x_3$ .



Turn into aij weights via softmax

Divide score by  $\sqrt{d_k}$ 

Compare  $x_3$ 's query with the keys for  $x_1$ ,  $x_2$ , and  $x_3$ .

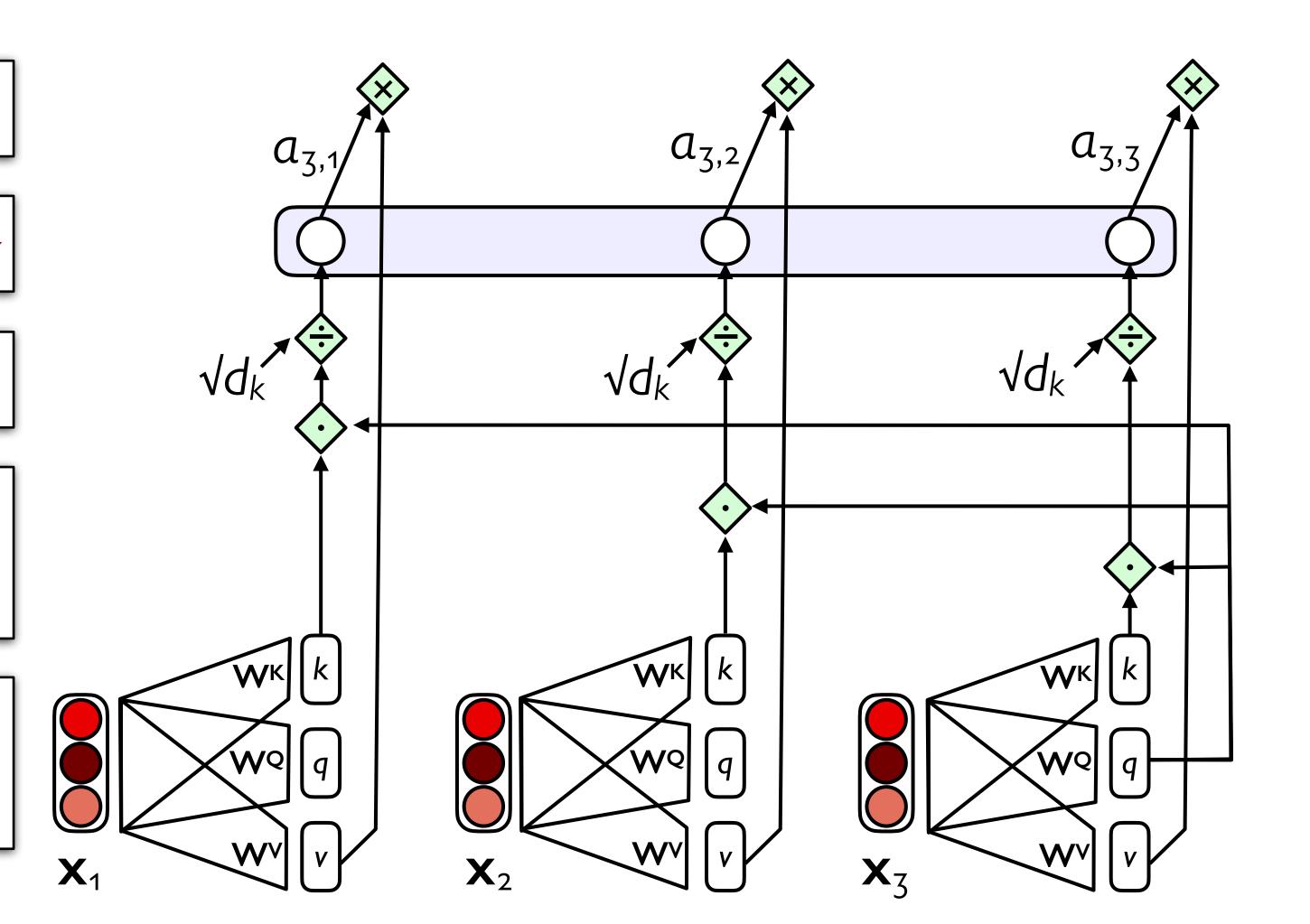


Weigh each value vector

Turn into aij weights via softmax

Divide score by  $\sqrt{d_k}$ 

Compare  $x_3$ 's query with the keys for  $x_1$ ,  $x_2$ , and  $x_3$ .



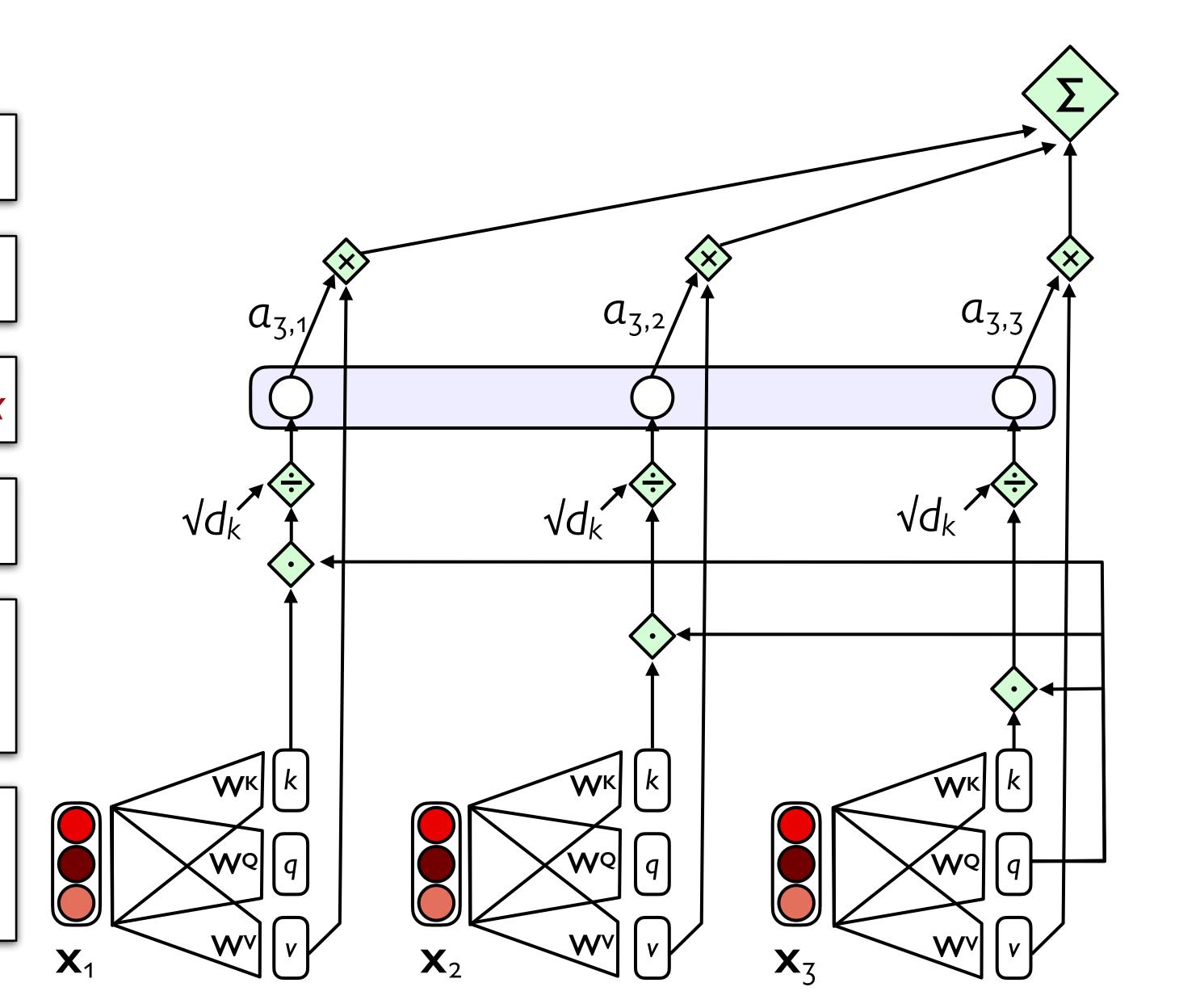
Sum the weighted value vectors

Weigh each value vector

Turn into aij weights via softmax

Divide score by  $\sqrt{d_k}$ 

Compare  $x_3$ 's query with the keys for  $x_1$ ,  $x_2$ , and  $x_3$ .



#### Output of self-attention

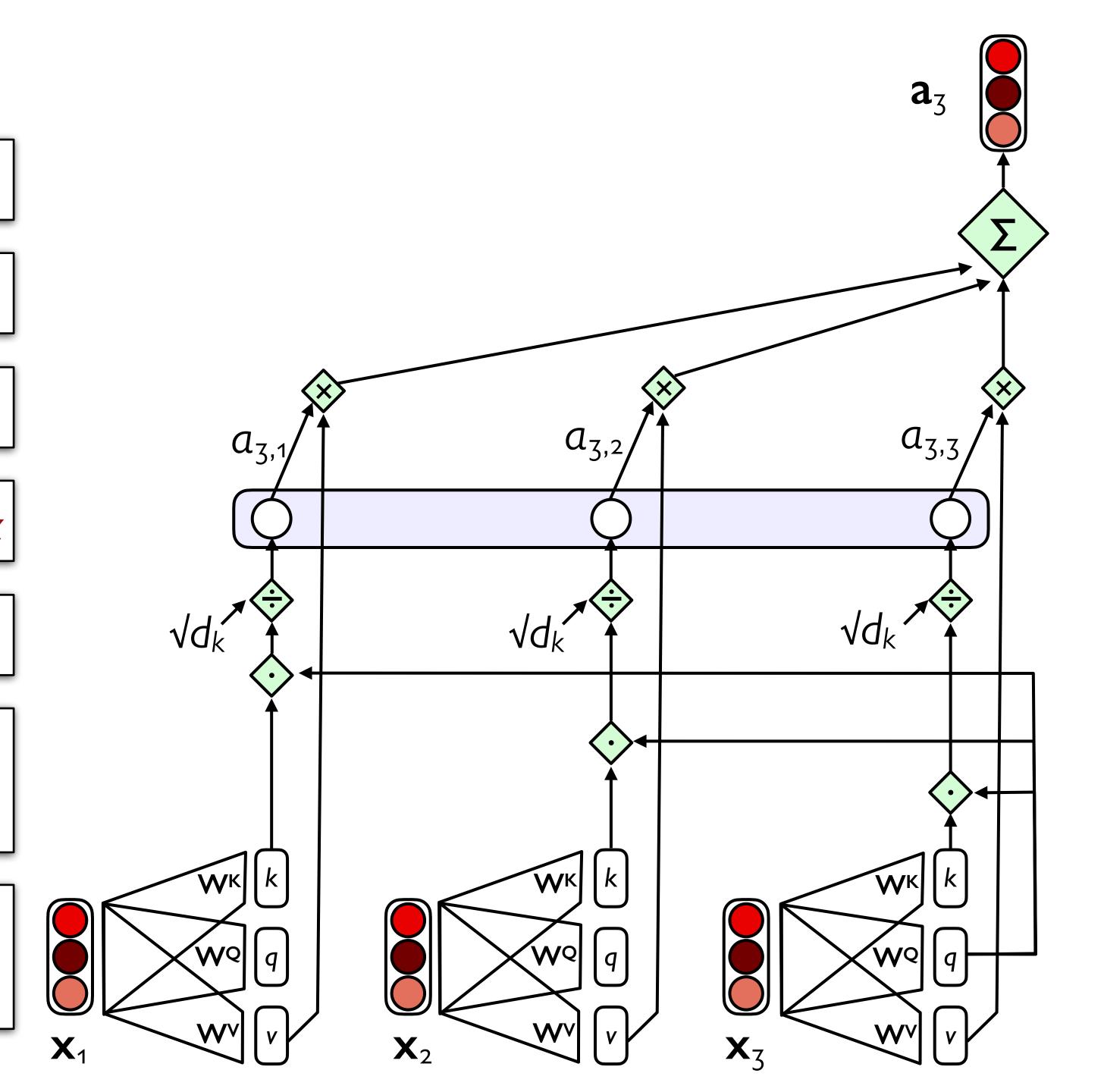
Sum the weighted value vectors

Weigh each value vector

Turn into aij weights via softmax

Divide score by  $\sqrt{d_k}$ 

Compare  $x_3$ 's query with the keys for  $x_1$ ,  $x_2$ , and  $x_3$ .







They don't tell you this in the paper (well they do but you have to read it like 15 times)



Multiplying
a lot of vectors
a lot of times
with scaled softmax



6:20 PM · Feb 22, 2023 · **88.3K** Views

In practice, instead of one attention head, we'll have lots of them: *multi-head attention*.

Why? Each head might be attending to the context for different purposes — different linguistic relationships or patterns in the context

## Summary

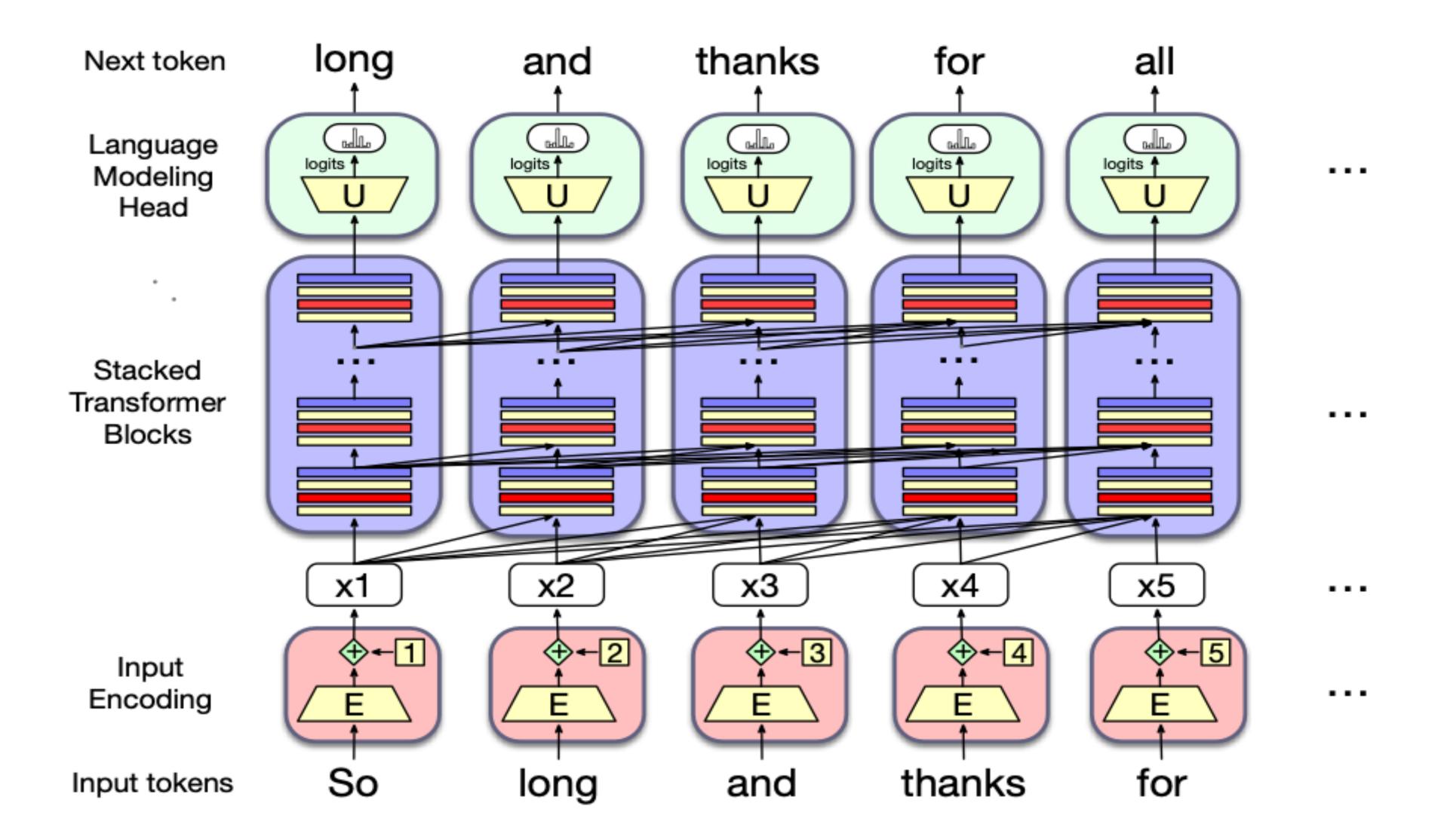
Attention is a method for enriching the representation of a token by incorporating contextual information

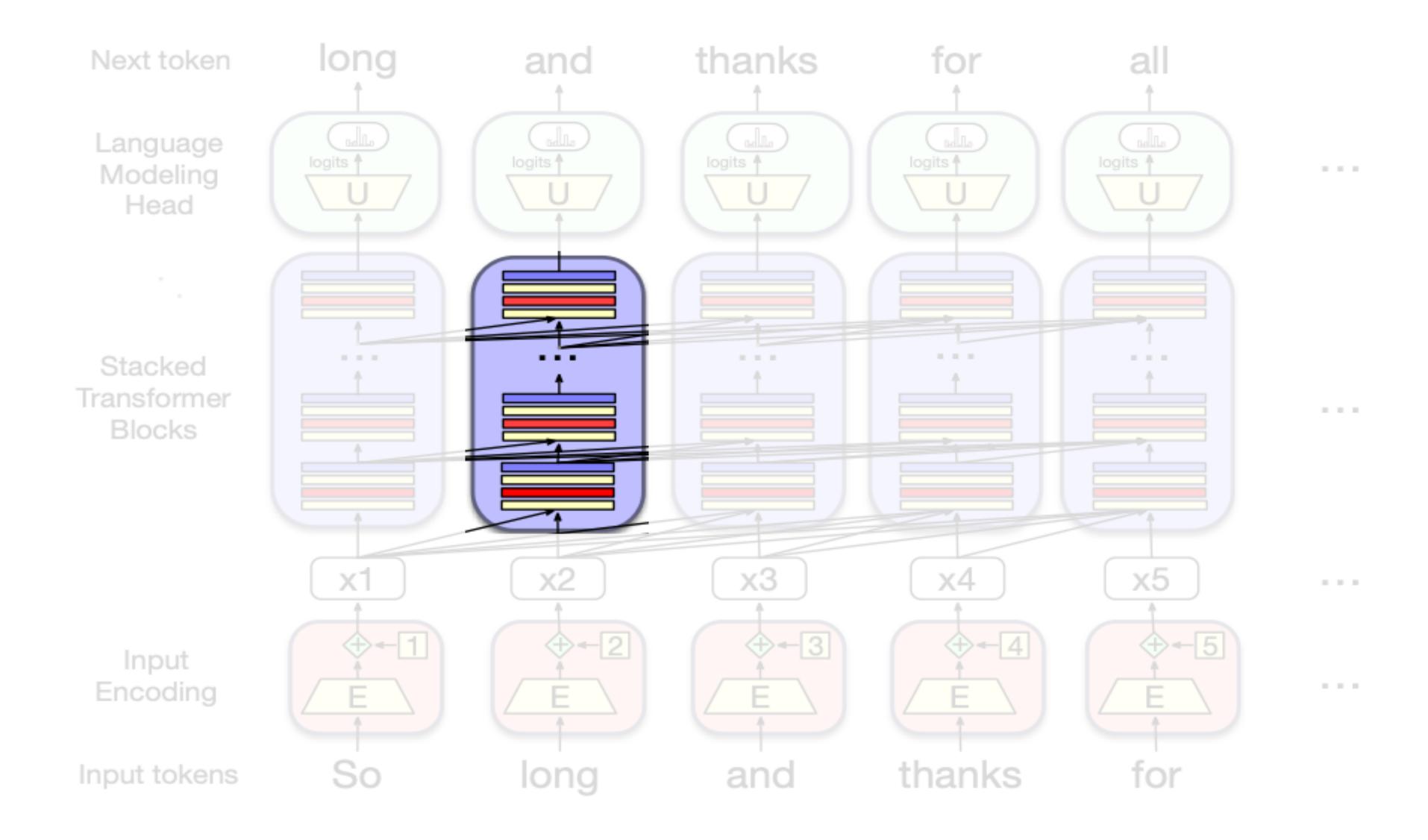
The result: the embedding for each word will be different in different contexts!

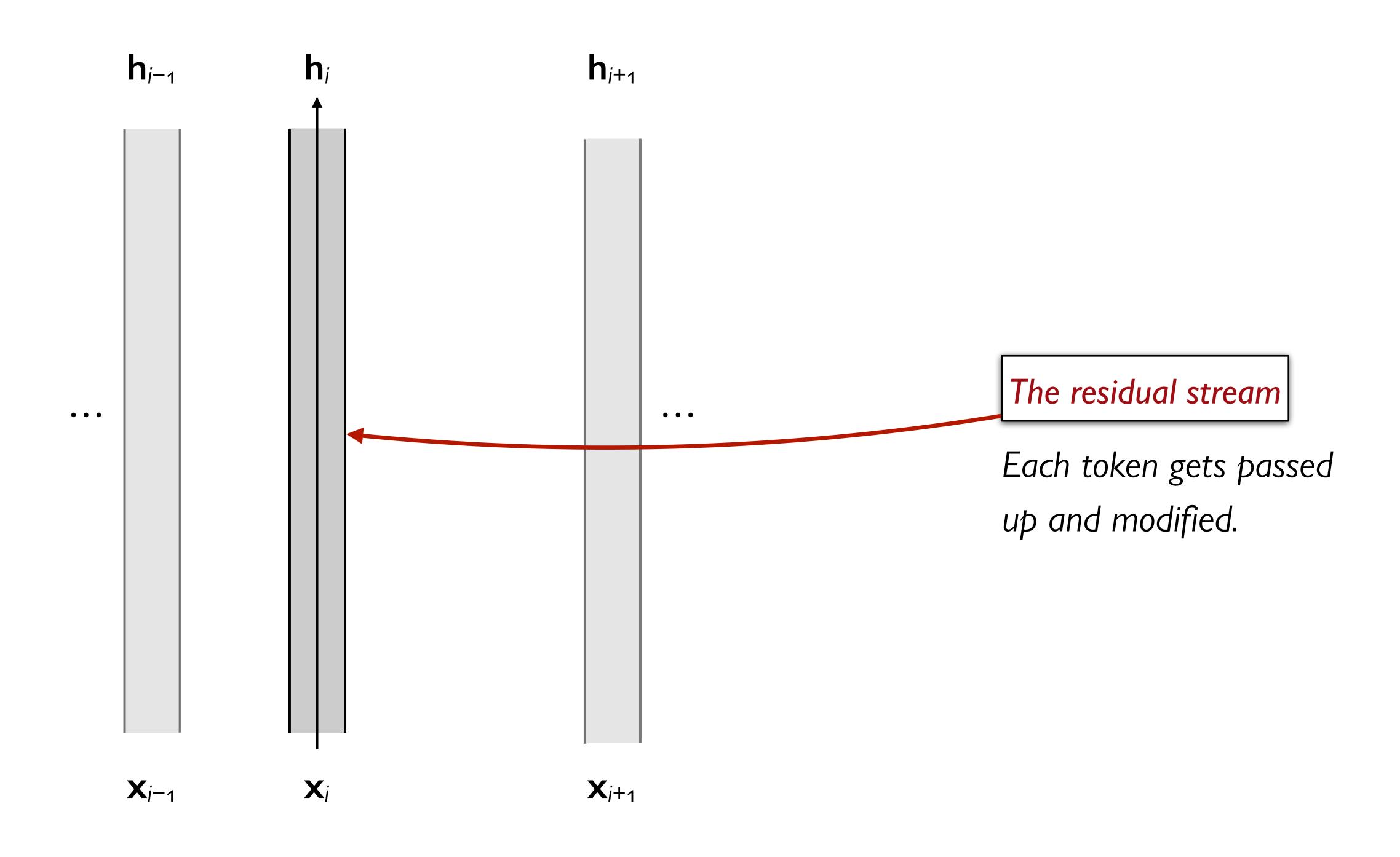
Contextual embeddings: a representation of word meaning in its context.

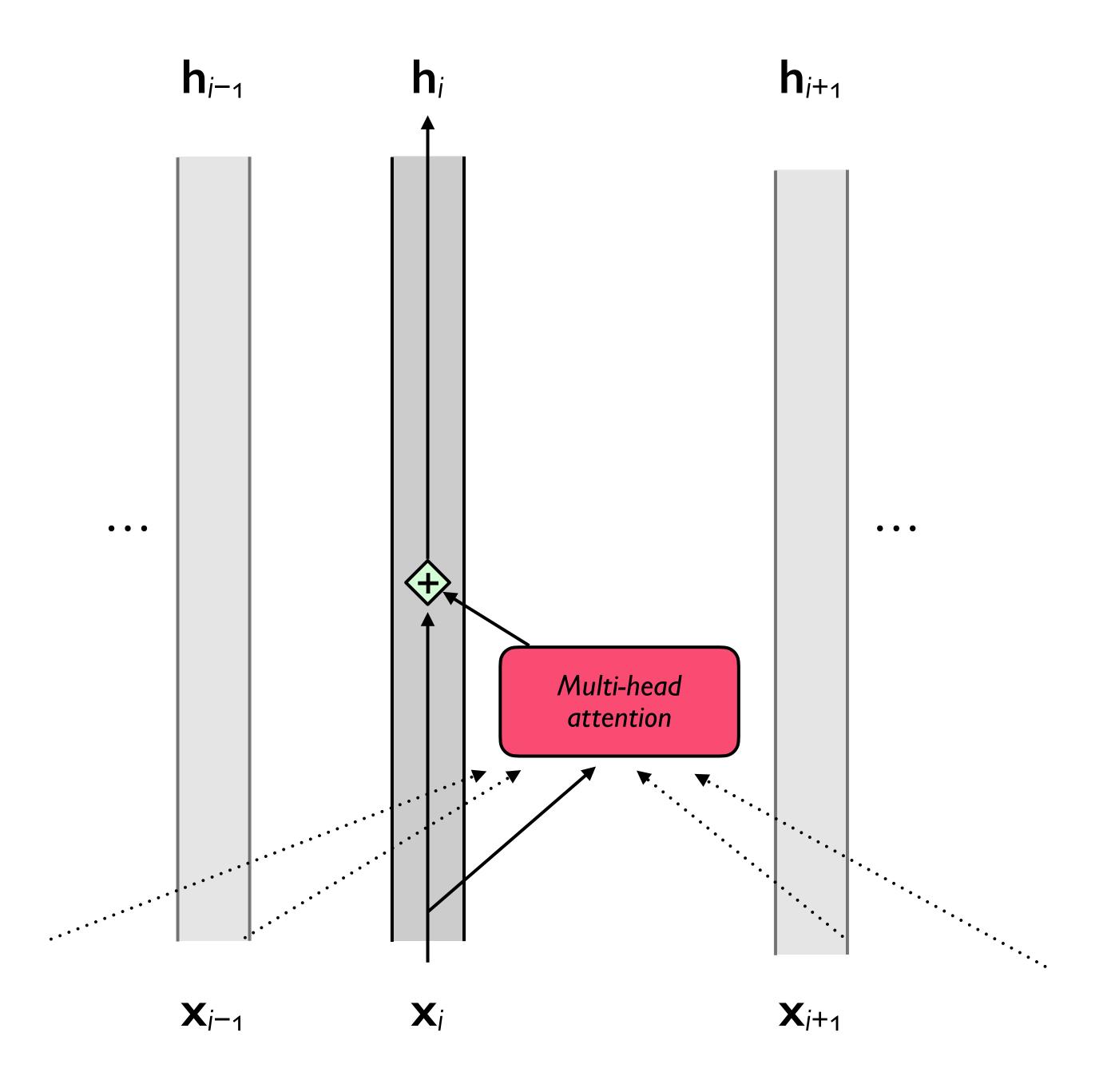
We'll see next that attention can also be viewed as a way to move information from one token to another.

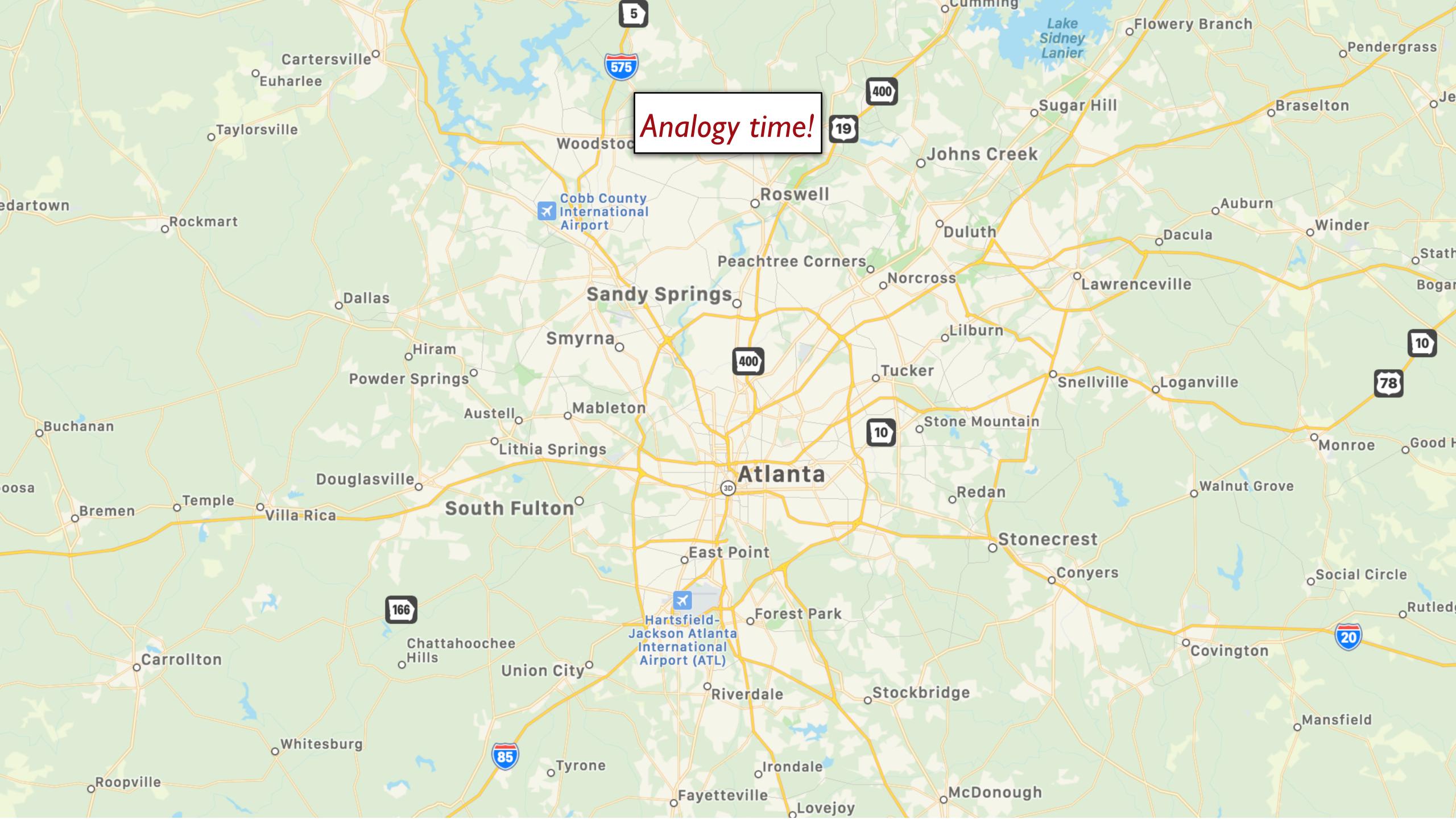
# The transformer block

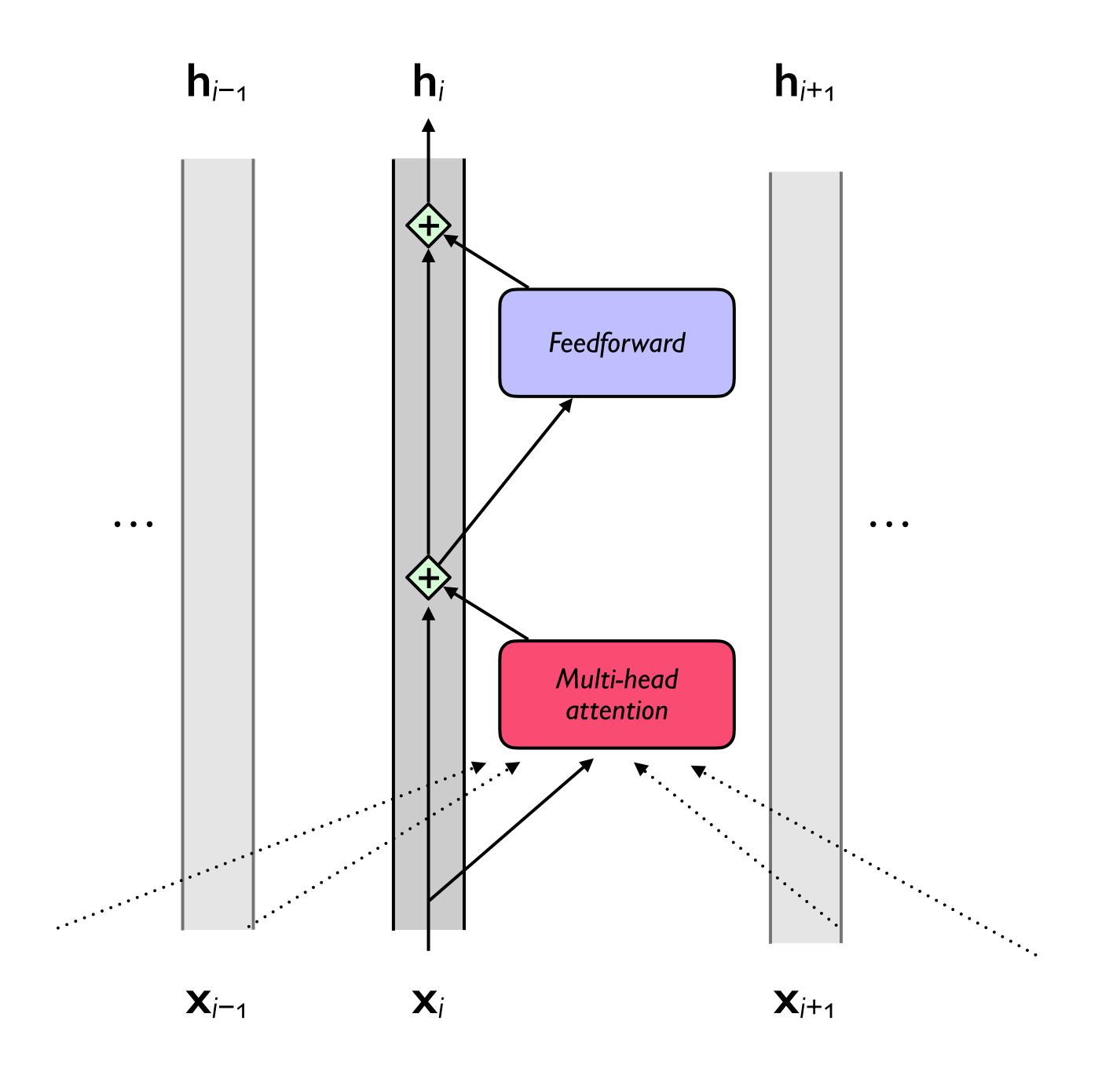






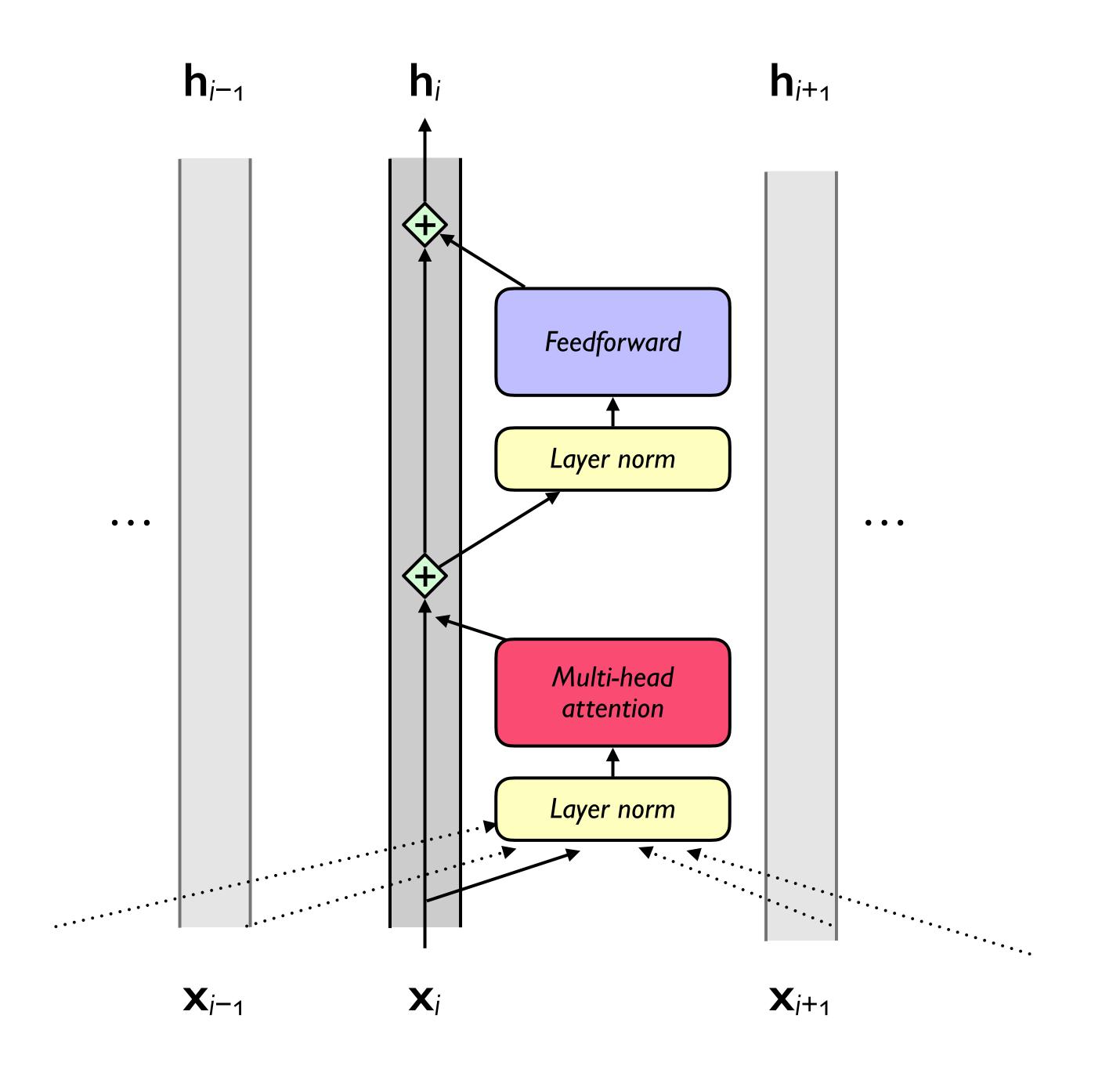






We'll need nonlinearities, so add a feedforward layer

$$FFN(\mathbf{x}_i) = ReLU(\mathbf{x}_i\mathbf{W}_1 + b_1)\mathbf{W}_2 + b_2$$



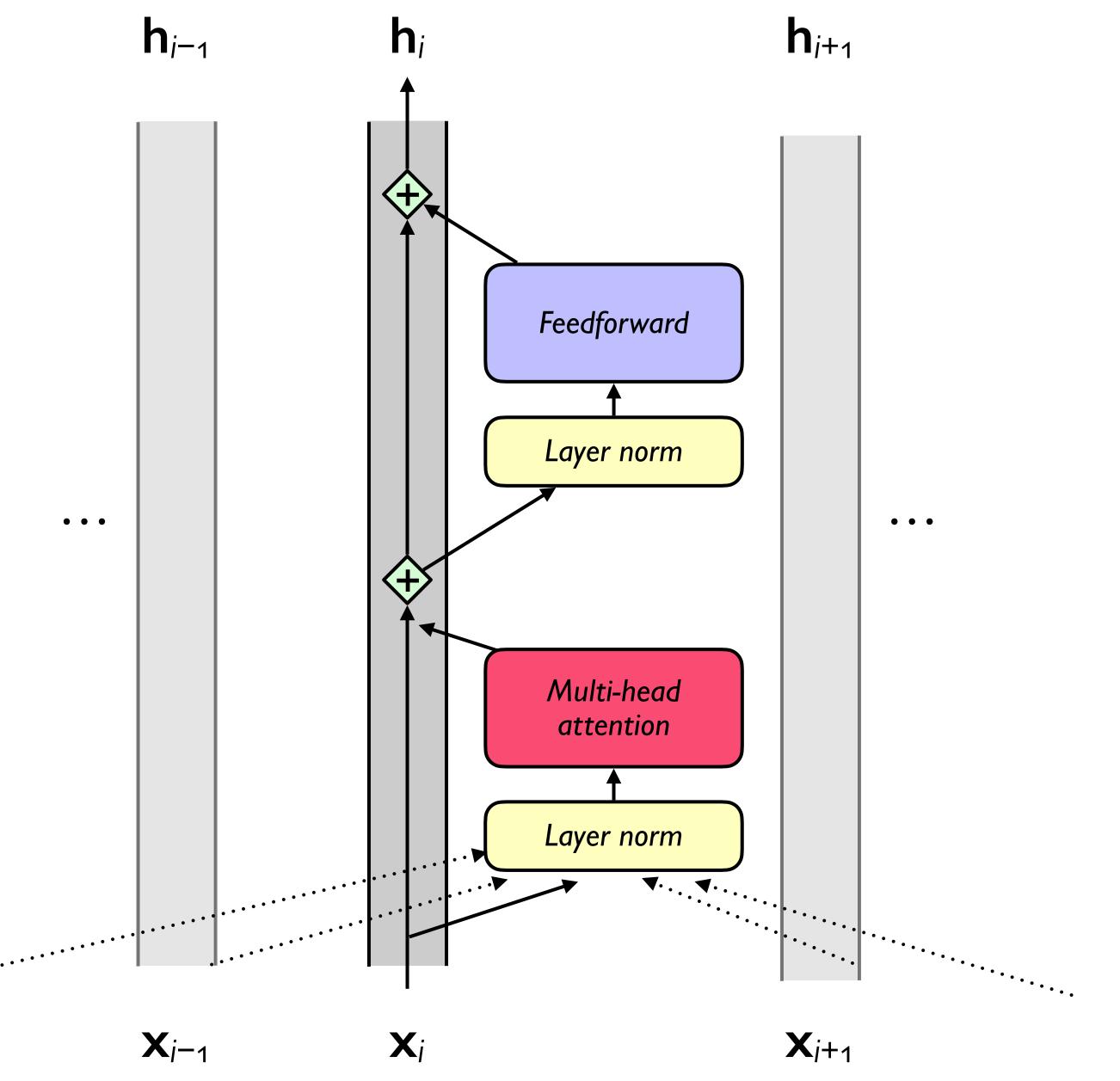
Layer norm

The vector  $\mathbf{x}_i$  is normalized twice

Layer norm is a variation of the z-score from statistics, applied to a single vector in a hidden layer:

 $\sigma = \sqrt{\frac{1}{d} \sum_{i=1}^{d} (x_i - \mu)^2}$ standard deviation  $\mathbf{\hat{x}} = \frac{(\mathbf{x} - \boldsymbol{\mu})}{-}$ normalized LayerNorm( $\mathbf{x}$ ) =  $\gamma \frac{(\mathbf{x} - \mu)}{\sigma} + \beta$ 

normalized in a tunable way



$$\mathbf{h}_i = \mathbf{t}_i^5 + \mathbf{t}_i^3$$

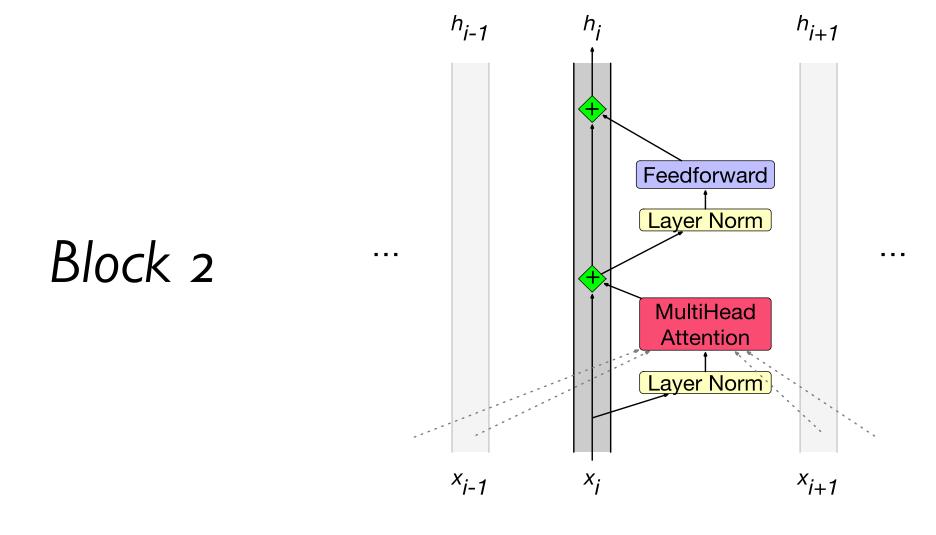
$$\mathbf{t}_i^5 = \text{FFN}(\mathbf{t}_i^4)$$

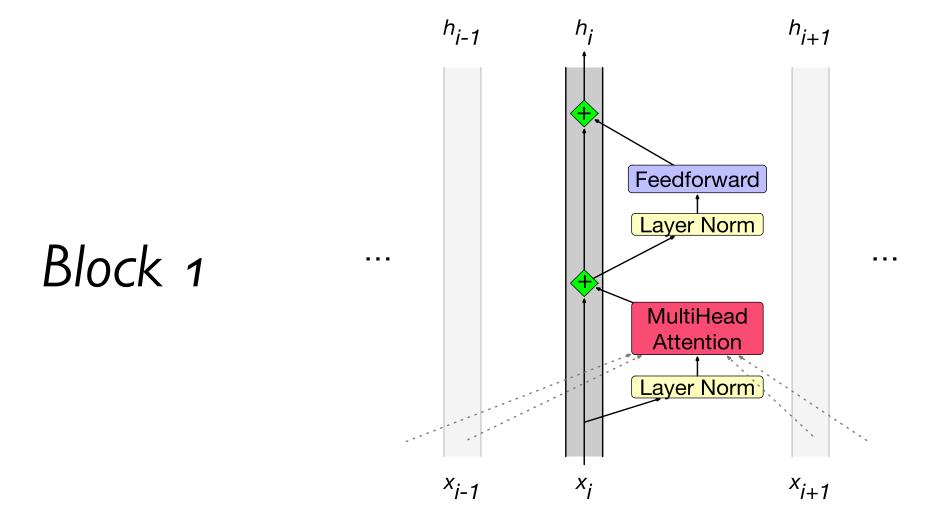
$$\mathbf{t}_i^4 = \text{LayerNorm}(\mathbf{t}_i^3)$$

$$\mathbf{t}_i^3 = \mathbf{t}_i^2 + \mathbf{x}_i$$

$$\mathbf{t}_i^2 = \text{MultiHeadAttention}(\mathbf{t}_i^1, [\mathbf{t}_1^1, ..., \mathbf{t}_N^1])$$

$$\mathbf{t}_i^1 = \text{LayerNorm}(\mathbf{x}_i)$$

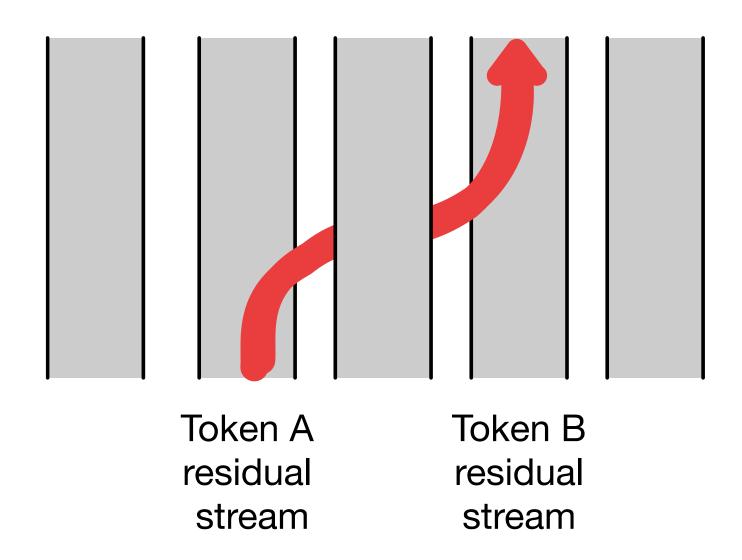




A transformer is a stack of these blocks — so all the vectors are of the same dimensionality d

Notice that all parts of the transformer block apply to 1 residual stream (1 token) – except attention, which takes information from other tokens.

Elhage et al. (2021) show that we can view attention heads as literally moving information from the residual stream of a neighboring token into the current stream:



# Acknowledgments

The lecture incorporates material from:

Jurafsky & Martin, Speech and Language Processing, 3rd ed. draft

