Vehicular Connectivity Models: From Single-Hop Links to Large-Scale Behavior

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***Abstract*—Focusing on large-scale vehicular ad-hoc networks (VANETs), we consider the interplay between single-hop channel models and large-scale network connectivity. Building on a realistic urban traffic simulator, we progressively increase the sophistication of the wireless link while evaluating the resulting connectivity profiles. Our results show that large-scale VANET connectivity, whose understanding is paramount towards the development of protocols and applications for this class of networks, is equally influenced by the choice of model and by the fine-tuning of its key parameters. Analyzing the distributions of both node degree and the duration of connection, we conclude that (a) as far as large-scale node degree behavior is concerned, a complex shadow fading environment is well approximated by a simpler and more tractable unit-disk model and, (b) unit-disk models allow longer connections than other models.**

***Index Terms*—VANET, Connectivity, Link models, Large-scale simulation**

I. Introduction

Realizing the vision of a Vehicular Ad-Hoc Network (VANET), in which wireless communication protocols enable safer, more efficient navigation and enhanced passenger satisfaction, requires a thorough understanding of the large-scale behavior of this class of distributed systems, particularly in what concerns the complex ways by which vehicles can connect and interact with each other. Because of the mathematical intractability of realistic urban traffic models with thousands of vehicles and the costs of real-life implementations, computer simulations combining vehicle mobility and wireless connectivity emerge as a valuable tool to evaluate the performance of VANET protocols and applications prior to their deployment in real systems.

When and how often two or more vehicles establish a reliable communication link depends mainly on (a) the traffic patterns of the road network in which they are immersed and (b) the propagation characteristics of the wireless medium they use to communicate. The effects of (a) on protocol performance have been studied extensively in the literature. In [1], [2], Barret compares the connectivity properties resulting from different spatial distributions and mobility models in simulations. Mobility models are further addressed in [3], [4], [5].

We focus on (b), and ask how the choice of single-hop wireless link model affects the large-scale connectivity of VANETs. To answer this question we make extensive use of the DIVERT framework (Development of Inter-VEhicular Reliable Telematics [6]), which allows for micro-simulation of thousands of vehicles with a high degree of realism.

Our main contributions are as follows:

- *Large-scale connectivity of VANETs:* We consider one-hop wireless link models with varying degrees of sophistication, ranging from unit disks to shadow fading environments, and evaluate their impact on both the standard and the transient connectivity of vehicular networks, when deployed in urban environments.
- *Fundamental Trade-offs:* We characterize the trade-offs between computational complexity and simulation accuracy, and show that depending on the metric of interest, complex models can, in some situations, be replaced by simpler ones without affecting the result.

Our work differs from related work on network connectivity and physical layer models for mobile ad-hoc networks, such as [7], [8], [9], [10], in that we address in detail the special characteristics of vehicular networks. A sophisticated wireless model for VANETs is presented in [11], which shows a convincing matching with experimental observation but does not address the large-scale behavior of a VANET. Reference [2] is narrower in scope in that it focuses on the accuracy of computational approximations while limiting itself to two-ray ground propagation. Previous work approached the issue of VANET connectivity [12], but did not analyze it in great detail and did not make any comparison between different link models.

The remainder of the paper is organized as follows. Section II presents the channel models under consideration and our notion of network connectivity. Section III describes the actual simulation implementation and Section IV presents our results. Section V concludes the paper.

II. Background

A. Connectivity Metrics

Throughout this paper we evaluate network connectivity based on the notion of node degree. For an undirected graph, the degree of a vertex is the number of edges adjacent to it. Thus, the average node degree is the average number of neighbors per node. In the case of mobile nodes in a VANET we extend the average node degree formulation to include not only the current links but also all connections established at previous time instants. This creates time-ordered paths that allow the exchange of older information, which is useful in a number of application domains. The average node degree for a time interval $[0...t]$ is then given by

$$d_{avg[0...t]} = \frac{1}{n}\sum_{i=1}^{n}\left|\bigcup_{j=0}^{t}\mathcal{N}(u_i)_j\right|,$$

where $\mathcal{N}(u_i)_j$ is the set of neighbors of node u_i at instant j.

Given the wireless broadcast characteristics of typical VANETs, it makes sense to also consider the information flows across multiple hops, which can be captured by a transitive closure graph. A node u can now connect with node v in one of two ways: either through a direct link as before or by communicating with a third node z which has established a connection with v at a previous time instant.

We base the transitive closure computation on an extension of the Floyd-Warshall algorithm [12]. The set of links for the transitive closure graph $[0...t]$ is a set of triples of the form $\{u_i, u_j, t'\}$, with $0 \leq t' \leq t$. The set of edges is written $\mathcal{L}' \supseteq \mathcal{U}^2 \times [0...t]$ and the transitive closure $\mathcal{L}'^*$ of $\mathcal{L}'$ can be computed for all $\forall u_i, u_j \in \mathcal{U}$ and $t' \in [0....t]$ according to

$$\{u_i, u_j, t'\} \in \mathcal{L}'^* \leftrightarrow \begin{cases} u_j \in \mathcal{N}(u_i)_{t'} \\ \exists u_k \in \mathcal{U}, \{u_i, u_k, t'\} \in \mathcal{L}' \wedge \\ \{u_k, u_j, t''\} \in \mathcal{L}'^* \wedge t'' < t'. \end{cases}$$

In other words, $\mathcal{L}'^*_{[0...t]}$ is the union of the set of all connections up to instant t with the set of all connections between u_i and u_j done through a third node u_k up to the same instant t. Recursively, this produces all paths of finite length.

B. Single-hop Channel Models

Ideally we would like to model the wireless channel between two vehicles with absolute accuracy. However, due to limitations in available computational power, radio propagation models in large-scale network research are subject to simplifying assumptions and the necessity of computable expressions.

1) Unit Disk Model: One of the most common and simplest models is the *Unit Disk Graph* model [13]. Here, nodes are located in the Euclidean plane and are assumed to have identical, normalized and unitary transmission radii forming a disk $D(P) = \{Q : |P - Q| < 1\}$ around node P. An edge is formed and communication can occur between two nodes whenever a node is contained in another's unit disk, i.e. the distance between them is less than unitary. All links are therefore undirected, assuming that if node u is able to hear node v, then node v can likewise communicate with node u.

While this model is simple and provides a tractable framework for analytical analysis, it fails to account for heterogeneity between nodes and obstacles, such as walls, buildings or weather conditions, all of which can affect the real-life transmission range. Also, the sharp cut-off at the disk's boundary fails to capture random noise and interference that can make even nearby nodes unreachable.

2) Quasi-Unit Disk Models: The *Quasi-Unit Disk Graph* model [14] is a generalization of the *Unit Disk Graph*. In a *Quasi-Unit Disk Graph*, two nodes are connected if their distance is less than or equal to d, d being a parameter between 0 and 1. Furthermore, if the distance between two nodes is greater than 1, there is no edge between them. In the range between d and 1 the existence of an edge is unspecified and can be defined as an arbitrary, implementation-dependent linear or non-linear function. We chose to evaluate 3 functions for this region: (a) linear fading where the connection probability decreases linearly with distance; (b) exponential fading with connection probability as an inverse exponential of the distance and (c) constant fading that assumes that all nodes at distance $d < x < 1$ have a 50% chance of being reached.

3) Shadow Fading Models: Reference [15] presents a more sophisticated model by considering the randomness induced by shadowing effects. Consider two nodes u and v at a relative distance $s(u, v)$. In this environment the attenuation $\beta(u, v)$ is given by the sum of two components, a distance-based component β_1 and a shadow fading component β_2. The deterministic distance-based term β_1 is given by

$$\beta_1(u, v) = \alpha\ 10\ \log\frac{s(u, v)}{1\ m}\ dB,$$

with pathloss exponent α a parameter usually between 1.5 and 6. In a log-normal shadow fading model, the second component β_2 in dB is given by a $(0, \sigma^2)$ normal distribution, with σ up to $12dB$.

For a given transmission power p_t and a minimum received power $p_{r,th}$, two nodes u and v can communicate if the attenuation between them is less than or equal to the threshold attenuation. Link probability is given by

$$P(\beta(u, v) \leq \beta_{th}|s(u, v)) = \frac{1}{2} - \frac{1}{2}\ \text{erf}\left(\frac{10\alpha}{\sqrt{2}\sigma}log\frac{s(u, v)}{r_0}dB\right),$$

with r_0 denoting the maximum distance achieving a link in the absence of fading ($\sigma = 0$).

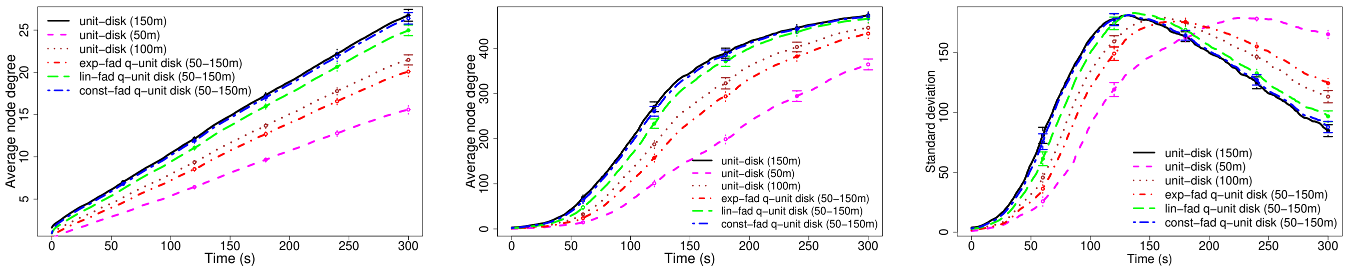
If we plot the link probability as a function of distance for this model we get a curve similar to an inverse exponential, as opposed to the square shape of the disk-based models.

III. Simulation Design

Our work is based on the in-house developed DIVERT VANET simulator with an extended link model layer. When compared with other alternatives [16], [17], [18], the support for real-life city maps (see the demos at [19]) and large-scale simulations make DIVERT ideal for our purposes.

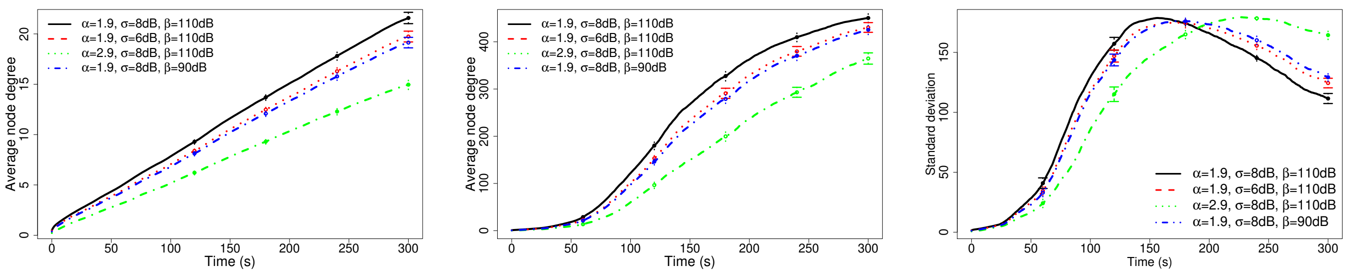
We used a map of the city of Porto with 965 Km of roads spread across an area of 75 Km^2, running a total of 200 simulations with 10.000 simultaneous vehicles, 10% of which wireless-enabled. We opted for $300s$-long simulations to study the transient behavior of the node degree distribution.

Vehicles follow two types of routes: shortest-distance routes between random points A and B and user-defined routes that match real life traffic patterns. Vehicles can enter and exit



(a) Average node degree (b) Average transitive node degree (c) Transitive standard deviation

Fig. 1. Node degree results for the disk models



(a) Average node degree (b) Average transitive node degree (c) Transitive standard deviation

Fig. 2. Node degree results for the shadow models

the simulation at the map's boundaries. These vehicles are therefore discarded, leaving around 500 stable vehicles for our analysis. DIVERT simulates each vehicle independently according to a car-following model [20]. In this model, vehicles make adjustments to their speed as to keep a safe distance to the car in front, accelerating and braking as needed. In our urban scenario, this results in average speeds of around 35 Km/h after 300 seconds.

We used 10 different link models in our analysis: 50, 100 and $150m$ radius unit-disks; $50-150m$ quasi-unit disks with a choice of exponential, linear and constant fading for the outer region and a base $\alpha = 1.9, \sigma = 6, \beta_{th} = 110dB$ shadow model plus 3 variations changing one parameter at a time from the set $\{\alpha = 2.9, \sigma = 8, \beta_{th} = 90dB\}$.

The values for the unit-disk radii were chosen in order to validate the values presented in [12]. The shadow fading parameters were chosen based on previous experimental results, the transmit power of $33dBm$ specified by the Car-2-Car Consortium [21] and typical wireless board specifications. In [22], an $\alpha = 2.8, \sigma = 6$ shadow model is shown to closely match the observed packet delivery ratio in an experiment with 33 nodes moving in an open-field. This was confirmed by our own field experiment with 12 laptops simulating a typical situation where vehicles exchange information while stopped at a traffic light, which shall be described in a future paper.

A. Computation Speed-Up

Computing the links every second is computationally expensive if we resort to the naÔve algorithm of testing every car against each other, which yields $O(n^2)$ cyclomatic complexity on the number of vehicles. Through optimization, we reduce this cost to $O(n)$. First, the road network is discretized into a 2D-grid. Then, a pre-processor goes through every cell and computes the set of cells that can be reached from that location with non-trivial probability. Now, at runtime, each car only needs to test the cells that are reachable from its current position, resulting in a linear cyclomatic complexity algorithm for all practical purposes. The code used in the simulations can be found at www.dcc.fc.up.pt/~rui.meireles/dump/fdmcode.zip.

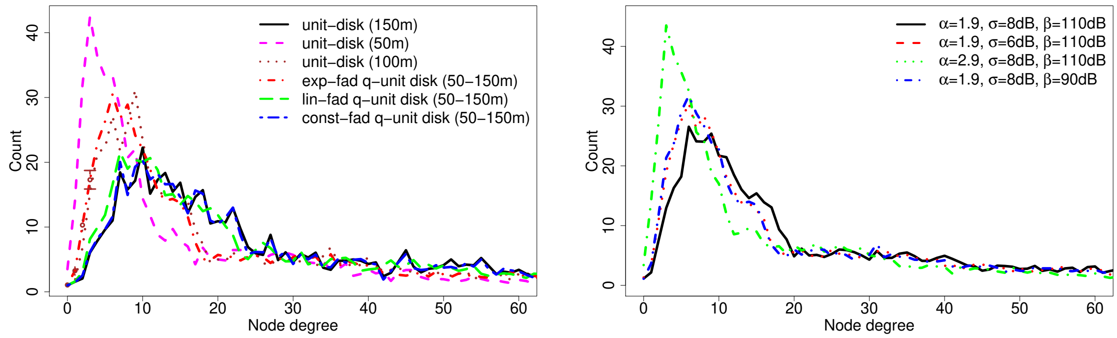
IV. Results

A. Node Degree

First we analyze the cumulative average node degree for the disk models—Figure 1. Boxes represent the 95% confidence interval at selected time stamps.

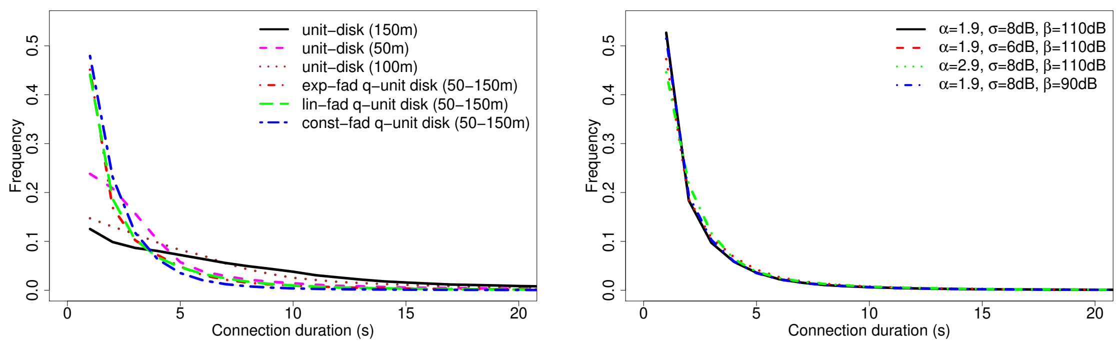
The non-transitive average node degree—Figure 1(a)—increases linearly with time for every model. The $50m$ unit-disk averages around 15 neighbors after 300 seconds. Increasing the radius to $100m$ yields 5 more neighbors per vehicle, as does increasing it again to $150m$. This result indicates that while most connections are established at short distances, increasing the transmission radius impacts connectivity significantly.

In the quasi-unit disk models, the effect of the three different outer region fading patterns can be clearly recognized. Constant fading with link probability 50% for the whole region gives almost the same connectivity as the $150m$ unit-disk, indicating that vehicles have usually more than one chance to



(a) Disk models (b) Shadow models

Fig. 3. Node degree distribution



(a) Disk models (b) Shadow models

Fig. 4. Connection duration distribution

communicate with their neighbors—the geometric distribution with $P = 0.5$ gives more than a 95% chance of success for 5 transmission attempts. Connectivity from the linear fading variant is close to the constant fading variant and exponential fading falls close to the $100m$ unit disk.

Including indirect connections reflects well on the average node degree, which increases by an order of magnitude—Figure 1(b). All models except the $50m$ unit-disk approach the 400 neighbors mark, meaning each node has, on average, received information from more than 80% of all wireless-equipped vehicles. The curve's relative positions remains the same as for the direct graph.

Please note that, unlike the case with only direct connections, the transitive node degree does not increase linearly at a constant rate with time. We can identify (a) a linear increase in the beginning when cars have not traveled long and links are mostly direct, (b) an intense burst around the $60s$ mark when previously isolated nodes receive condensed information from another node and (c) a final phase where the node degree slowly approaches the possible maximum.

We now consider the standard deviation of the cumulative node degree. In the direct graph case, we get a standard deviation that increases linearly with time and is of the same order of magnitude of the average node degree for all models (Figure omitted for lack of space). This indicates that the node degree distribution is not uniform. In the transitive closure graph case—Figure 1(c), we see that the standard deviation starts decreasing after a while, and sooner the more optimistic the model. This is a reflex of the percolation effect that happens when a node u with few connections links up with a node with a high node degree v and assimilates all of its information, thus greatly increasing the cardinality of its own set of transitive neighbors.

Analyzing the node degree for the shadow models in Figure 2 we observe that they behave in a very similar fashion to the disk-based models. The model with $\alpha = 1.9, \sigma = 8, \beta_{th} = 110dB$ behaves similarly to the $100m$ unit-disk. Reducing the acceptable attenuation from 110 to $90dB$ decreases connectivity slightly, the same happens when we decrease the noise standard deviation to $6dB$. Increasing the pathloss exponent α to 2.9 has a more severe effect, resulting in a behavior similar to a $50m$ unit-disk.

Plotting the node degree distribution for the direct graph after 300 seconds—Figure 3, we confirm that it is nonuniform. While most nodes have a number of neighbors close to the average value, there is a long right-tail that leads to a high standard deviation as previously discussed. As before, we can find an almost perfect correspondence between the disk and the shadow models.

B. Connection duration

Thus far we have seen that, in terms of node degree, there is no significative difference between using a shadow model over an appropriately sized unit-disk. We now check if this holds true for another important metric, the amount of time that a connection between two vehicles is kept unbroken. Figure 4 plots the connection duration distribution with the 1-second resolution allowed by our simulation platform. In all models, most connections only last a few seconds. However, there is a clear distinction to be made between the unit-disk models and the quasi-unit disk and shadow models. The curves for the latter models resemble an inverse exponential, with much more unstable connections when compared with the unit-disk models. This is justified by the probabilistic nature of the shadow fading and quasi-unit disk models.

V. Conclusions

Convinced of the importance of understanding the large-scale connectivity of VANETs, we extended the functionalities of the DIVERT platform to characterize how the choice and calibration of a suitable single-hop channel model can impact the node degree and connection duration.

To compare the various candidate channel models, we carried out an extensive set of simulations. Our results concerning node degree show that (a) the network connectivity for a given model is heavily affected by the choice of parameters and that (b) a shadow fading model can easily be replaced by an appropriate unit-disk model without affecting the node degree distribution significantly. Our connection duration study tells a different tale, however. Results indicate that unit-disks allow for longer connections than either quasi-unit disks or shadow fading. This is important for the evaluation of several classes of algorithms, including routing algorithms based on path discovery and maintenance.

Finally, it is worth pointing out that our methodology is not specific to any one city or network. In fact the DIVERT framework can support other roadmaps, vehicle densities and choice of routes. Understanding the impact of the urban topology on the large-scale connectivity profiles is a compelling avenue for future research.

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