Problem Solving and Abstraction (CMPU 101)

Tom Ellman

Lecture 11

How would we write a function that takes a list of numbers and returns its sum?

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
  my-sum([list: 1, 4]) is 1 + 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
  my-sum([list: 1, 4]) is 1 + 4
  my-sum([list: 4]) is 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
  my-sum([list: 1, 4]) is 1 + 4
  my-sum([list: 4]) is 4
  my-sum([list: ]) is ...?...
end
```

The empty list can be written both as:

[list:]

and, a bit less awkwardly, as:

empty

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
  my-sum([list: 1, 4]) is 1 + 4
  my-sum([list: 4]) is 4
  my-sum([list: ]) is ?
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
  my-sum([list: 1, 4]) is 1 + 4
  my-sum([list: 4]) is 4
  my-sum([list: ]) is 0
end
```

Shouldn't:

my-sum(L.append(empty, [list: 3 1 4]))

be the same as:

my-sum(empty) + my-sum([List: 3 1 4])

The Secret Nature of Lists

Writing our input as [list: 3, 1, 4] hides the truth.

It's just a shorthand for the real structure of a list.

In its secret heart, Pyret knows there are only two ways of making a list:

The value: **empty**.

Using the **link** function to add an element to the beginning.

When we write an expression like:

[list: 3, 1, 4]

Pyret translates it into this:

```
link(3,                    ==> [list: 3, 1, 4]
  link(1,                  ==> [list: 1, 4]
    link(4, empty)))       ==> [list: 4]
```

Consider a non-empty list like:

[list: <first> …?...]

Pyret translates it into something like this:

link(<first>, <rest>)

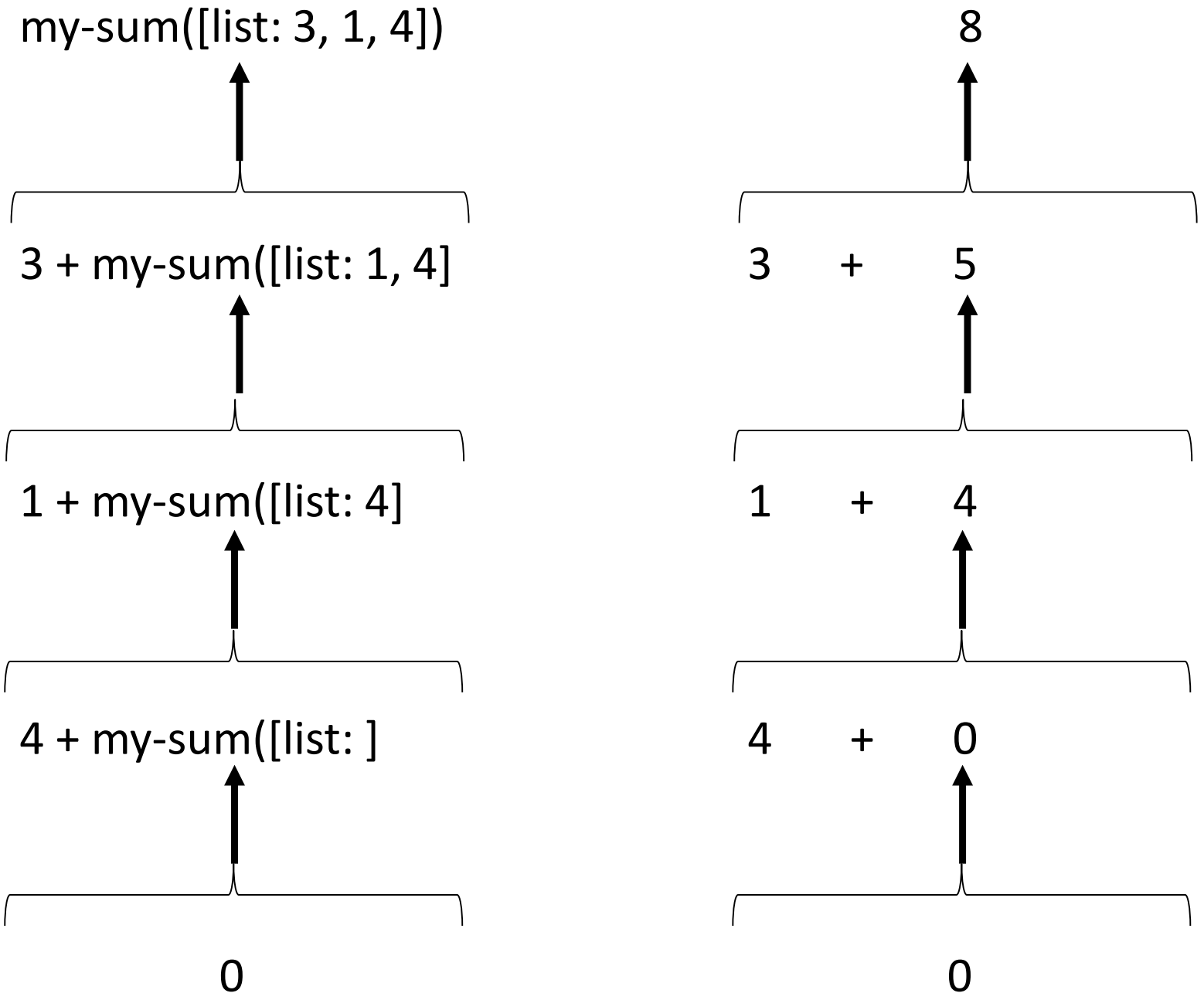
Where:

- <first> is any number.
- <rest> could be any list of numbers.

What can we say about: my-sum(link(<first>, <rest>)) ?

The sum is: <first> + my-sum(<rest>).

```
fun my-sum(lst :: List<Number>) -> Number:
  ...
end
where:
 my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
 my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
 my-sum([list: 4]) is 4 + my-sum([list: ])
 my-sum([list: ]) is 0
end
```


my-sum([list: 3, 1, 4])
3 + my-sum([list: 1, 4]
1 + my-sum([list: 4]
4 + my-sum([list:]
0
8
3 + 5
1 + 4
4 + 0
0

case


```
cases (List) lst:
  | empty => … ? …                  # Result if lst is empty.
  | link(first,rest) => … ? …       # Result if lst is not empty.
end
```

A **cases** expression is like an **if expression.** If the list is empty, do one thing. If it's a link, do another thing.

```
fun my-sum(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => … ? …
  | link(first,rest) => … ? …
end
where:
 my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
 my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
 my-sum([list: 4]) is 4 + my-sum([list: ])
 my-sum([list: ]) is 0
end
```

A **cases** expression is like an **if expression.** If the list is empty, do one thing. If it's a link, do another thing.

```
fun my-sum(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 0
  | link(first, rest) => first + my-sum(rest)
end
where:
 my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
 my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
 my-sum([list: 4]) is 4 + my-sum([list: ])
 my-sum([list: ]) is 0
end
```

When we call this function, it evaluates as:

my-sum(link(3, link(1, link(4, empty))))

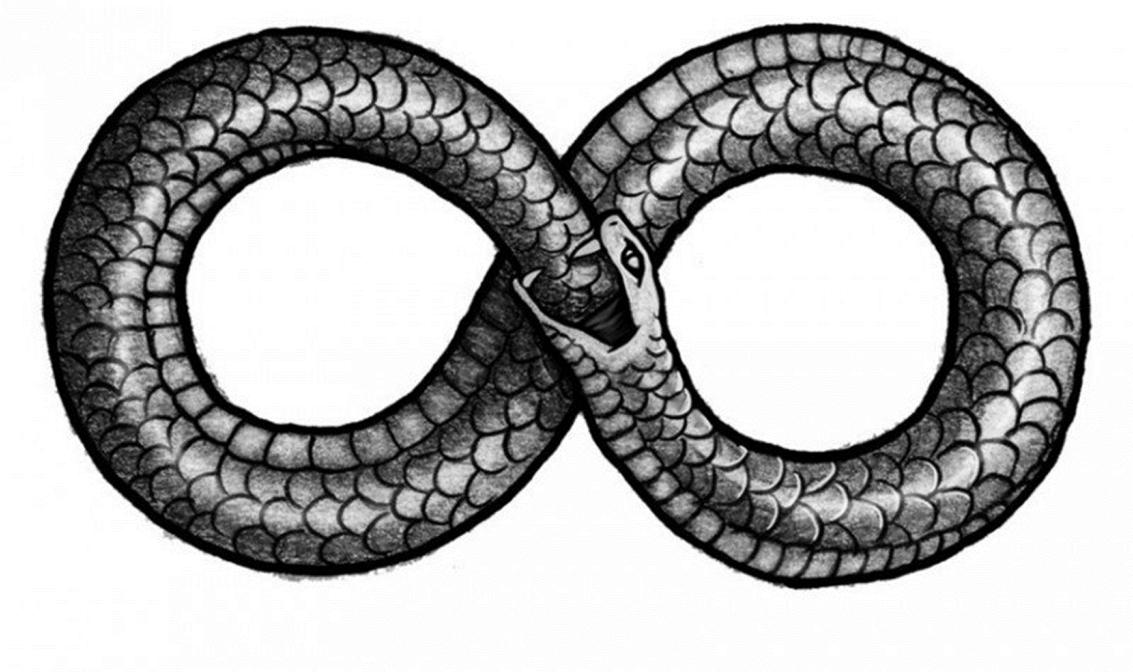
3 + my-sum(link(1, link(4, empty)))

3 + 1 + my-sum(link(4, empty))

3 + 1 + 4 + my-sum(empty)

3 + 1 + 4 + 0

When we call my-sum, the result it returns depends on other calls to my-sum!



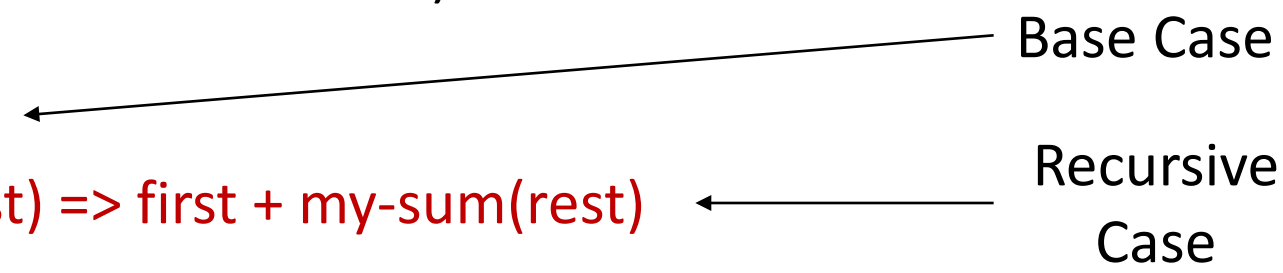
Can this really work?
Yes, but!

The sequence of recursive invocations of:

my-sum(<non-empty-list>)

must eventually lead to an invocation of **my-sum(empty)**.

```
fun my-sum(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 0
  | link(first, rest) => first + my-sum(rest)
end
where:
 my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
 my-sum([list: 1, 4])    is 1 + my-sum([list: 4])
 my-sum([list: 4])       is 4 + my-sum([list: ])
 my-sum([list: ])        is 0
end
```



Notice that **my-sum(lst)** leads to **my-sum(rest)**, where **rest** is one shorter than **lst**. So eventually we reach **my-sum(empty)** which is just zero.

Now that we have a definition of **my-sum**:

```
fun my-sum(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 0
  | link(first, rest) => first + my-sum(rest)
end
where:
  my-sum([list: ]) is 0
  my-sum([list: 3, 1, 4]) is 8
end
```

Maybe we can define a similar, and perhaps easier function: **my-product**.

```
fun my-product(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => … ? …
  | link(first,rest) => … ? …
end
where:
 my-product([list: 3, 1, 4]) is 3 * 1 * 4
 my-product([list: 1, 4]) is 1 * 4
 my-product([list: 4]) is 4
 my-product([list: ]) is …?...
end
```

Consider:

my-product(L.append(empty, [list: 1 2 3]))

Shouldn't it be the same as:

my-product(empty) * my-product([List: 1 2 3])

```
fun my-product(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => … ? …
  | link(first,rest) => … ? …
end
where:
 my-product([list: 3, 1, 4]) is 3 * my-product([list: 1, 4])
 my-product([list: 1, 4]) is 1 * my-product([list: 4])
 my-product([list: 4]) is 4 * my-product([list: ])
 my-product([list: ]) is 1
end
```

Consider:

my-product(L.append(empty, [list: 1 2 3]))

Shouldn't it be the same as:

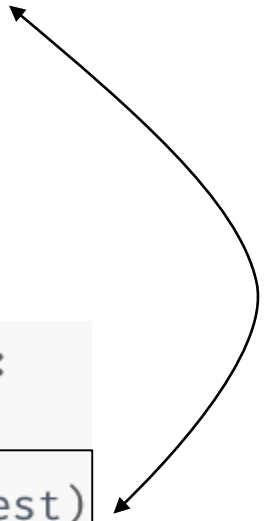
my-product(empty) * my-product([List: 1 2 3])

```
fun my-product(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 1
  | link(first,rest) => first * my-product(rest)
end
where:
 my-product([list: 3, 1, 4]) is 3 * my-product([list: 1, 4])
 my-product([list: 1, 4]) is 1 * my-product([list: 4])
 my-product([list: 4]) is 4 * my-product([list: ])
 my-product([list: ]) is 1
end
```

Now that we have definitions of **my-sum** and **my-product**:

```
fun my-sum(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 0
  | link(first, rest) => first + my-sum(rest)
end
where:
  my-sum([list: ]) is 0
  my-sum([list: 3, 1, 4]) is 8
end
```

```
fun my-product(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 1
  | link(first, rest) => first * my-product(rest)
end
where:
  my-product([list: ]) is 1
  my-product([list: 1,2,3,4,5]) is 120
end
```



Maybe we can define a similar, and perhaps easier function: **my-length**.

```
fun my-length(lst :: List<Any>) -> Number:
cases (List) lst:
    | empty => …?...
    | link(first, rest) => …?...
end
where:
 my-length([list: 3, 1, 4]) is 1 + my-length ([list: 1, 4])
 my-length([list: 1, 4])    is 1 + my-length ([list: 4])
 my-length([list: 4])       is 1 + my-length ([list: ])
 my-length([list: ])        is 0
end
```

```
fun my-length(lst :: List<Any>) -> Number:
cases (List) lst:
    | empty => 0
    | link(first, rest) =>  1 + my-length(rest)
end
where:
 my-length([list: 3, 1, 4]) is 1 + my-length ([list: 1, 4])
 my-length([list: 1, 4])    is 1 + my-length ([list: 4])
 my-length([list: 4])       is 1 + my-length ([list: ])
 my-length([list: ])        is 0
end
```

```
fun my-length(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 0
    | link(first, rest) => 1 + my-length(rest)
end
where:
  my-length([list: ]) is 0
  my-length([list: 3, 1, 4, 5]) is 4
end
```

```
fun my-increments(lst :: List<Number>, n :: Number)
-> List<Number>:

cases (List) lst:
| empty =>  ?
| link(first, rest) => ?
end
where:   my-increments(empty,7) is empty
         my-increments([list: 1,2,3],2) is [list: 3,4,5]
end
```

```
fun my-increments(lst :: List<Number>, n :: Number)
 -> List<Number>:

cases (List) lst:
| empty =>  empty
| link(first, rest) => link(n + first, my-increments(rest,n))
end
where:   my-increments(empty,7) is empty
         my-increments([list: 1,2,3],2) is [list: 3,4,5]
end
```

```
fun my-increments(lst :: List<Number>, n :: Number) -> List<Number>:
 cases (List) lst:
    | empty => empty
    | link(first, rest) => link(n + first, my-increments(rest,n))
 end
where:
  my-increments(empty,7) is empty
  my-increments([list: 1,2,3],2) is [list: 3,4,5]
end
```