Problem Solving and Abstraction
(CM PU 101)

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Lecture 12
The Secret Nature of Lists

Writing our input as [list: 3, 1, 4] hides the truth.

It’s just a shorthand for the real structure of a list.

In its secret heart, Pyret knows there are only two ways of making a list:

The value: `empty`.

Using the `link` function to add an element to the beginning.
When we write an expression like:

[list: 3, 1, 4]

Pyret translates it into this:

link(3,
    link(1,
        link(4, empty)))
Something that often trips people up when writing functions like this is the difference between:

```python
  link(x, y)
  and
  [list: x, y]
```

What happens if we change the former to the latter?

```python
>>> x = 1
>>> y = [list: 2, 3, 4]
>>> link(x, y)
[list: 1, 2, 3, 4]
>>> [list: x, y]
[list: 1, [list: 2, 3, 4]]
```
The second argument of link must be a (possibly empty) list.

```
link(<anything>, <list>)
```

```
>> link(1, 2)

The annotation, List, at builtin://lists:79:29-79:36 was not satisfied by the value

2
(Show program evaluation trace...)
```
Recursion is appropriate, any time a problem can be split into parts, one of which is a smaller version of the original problem.

\[
\text{sum(link(3, }
\text{ link(1,}
\text{ link(4, empty))})
\]

\[
3 + \text{sum(link(1,}
\text{ link(4, empty))))
\]
fun my-sum(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => 0
  | link(first, rest) => first + my-sum(rest)
end
where:
my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
my-sum([list: 4]) is 4 + my-sum([list: ])  
my-sum([list: ]) is 0
end
When we call this function, it evaluates as:

\[
\text{my-sum(link(3, link(1, link(4, empty)))))}
\]

\[
3 + \text{my-sum(link(1, link(4, empty))))}
\]

\[
3 + 1 + \text{my-sum(link(4, empty))}
\]

\[
3 + 1 + 4 + \text{my-sum(empty)}
\]

\[
3 + 1 + 4 + 0
\]
Recursion

• All recursive functions have these two parts:
  • Base case(s):
    – What’s the simplest case to solve?
  • Recursive case(s):
    – What’s the relationship between the current problem and the answer to a slightly smaller problem?
    – You should be calling the function you’re defining here; this is referred to as a recursive call.
fun rec-fun(lst :: List<Number>) -> Number:
cases (List) lst:
  | empty => Base Case
    # Expression giving value of rec-fun(empty)
  | link(first, rest) => Recursive Case
    # Expression giving value of rec-fun(link(first,rest))
    # in terms of first and rec-fun(rest).
end
end
all-below

• Given:
  – a list (lst) of numbers
  – a number (bnd)

• Return:
  – true if all numbers on lst are below bnd
  – false otherwise.
Examples

fun all-below(lst :: List<Number>, bnd :: Number) -> Boolean:
    ...

where:
    all-below(empty, 0) is true
    all-below([list: 1,2,3,4,5], 5) is false
    all-below([list: 1,2,3,4,5], 6) is true
end

Notice that all-below(empty,0) is true. Why?
fun all-below(lst :: List<Number>, bnd :: Number) -> Boolean:
cases (List) lst:
   | empty => true
   | link(first, rest) => (first < bnd) and all-below(rest, bnd)
end
where:
   all-below(empty, 0) is true
   all-below([list: 1,2,3,4,5], 5) is false
   all-below([list: 1,2,3,4,5], 6) is true
end

Notice that all-below is true if lst is empty, regardless of bnd.

In this case we cay that all-below is *vacuously* true!
any-above

• Given:
  – a list (lst) of numbers
  – a number (bnd)

• Return:
  – true if at least one number on lst is above bnd.
  – false otherwise.
Examples

fun any-above(lst :: List<Number>, bnd :: Number) -> Boolean:

...

where:
    any-above(empty, 0) is false
    any-above([list: 1,2,3,4,5], 5) is false
    any-above([list: 1,2,3,4,5], 4) is true
end

Notice that any-above(empty, 0) is false. Why?
Notice that any-above is false if lst is empty, regardless of bnd.

In this case we say that any-above is **vacuously** false.
increasing

• Given a list of numbers:

• Return:
  – true if each number with a predecessor is larger than its predecessor.
  – false if at least one number has a predecessor and is not larger than its predecessor.
Examples

fun increasing1(lst :: List<Number>) -> Boolean:

...?...

where:
  increasing1(empty) is true
  increasing1([list: 1]) is true
  increasing1([list: 1, 2, 3]) is true
  increasing1([list: 1, 3, 2]) is false
  increasing1([list: 1, 3, 3]) is false

end

Why must the first two tests come out true?
fun increasing1(lst :: List<Number>) -> Boolean:
  cases (List) lst:
  | empty => true
  | link(first, rest) =>
    if (rest == empty): true
    else: (first < rest.first) and increasing1(rest)
  end
end

where:
  increasing1(empty) is true
  increasing1([list: 1]) is true
  increasing1([list: 1, 2, 3]) is true
  increasing1([list: 1, 3, 2]) is false
  increasing1([list: 1, 3, 3]) is false
end
In this definition, we refrain from using a cases statement and rely on if-else instead.

This works, but it’s cumbersome and hard to read.
In this definition, we refrain from using a cases statement and rely on logical operators or & and.

This works, but it’s also cumbersome and hard to read.
my-all

• Given a predicate (pred) and a list (lst).

• Return:
  – true if pred returns true for each element of lst.
  – false if pred returns false for at least one element of lst.
Examples

```haskell
fun my-all(pred :: Function, lst :: List<Any>) -> Boolean:
    ...
where:
    my-all(lam(x): false end, empty) is true
end
```

How can my-all be true on the empty list, if the lambda expression always returns false?
fun my-all(pred :: Function, lst :: List<Any>) -> Boolean:
  cases (List) lst:
    | empty => true
    | link(first, rest) => pred(first) and my-all(pred, rest)
  end
where:
  my-all(lam(x): false end, empty) is true
end
my-any

• Given a predicate (pred) and a list (lst).

• Return:
  – true if pred returns true for at least one element of lst.
  – false if pred returns false for each element of lst.
Examples

fun my-any(pred :: Function, lst :: List<Any>) -> Boolean:
...

where:
my-any(lam(x): true end, empty) is false
end

How can my-all be false on the empty list, if the lambda expression always returns true?
fun my-any(p: Function, l: List<Any>) -> Boolean:
  cases (List) l:
    | empty => false
    | link(first, rest) => p(first) or my-all(p, rest)
  end
where:
  my-any(lam(x): true end, empty) is false